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ON NON-HOMOGENEOUS TERNARY CUBIC DIOPHANTINE EQUATION

$$3x^2 + 4y^3 = z^2$$

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ABSTRACT

The non-homogeneous ternary cubic diophantine equation $3x^2 + 4y^3 = z^2$ is analyzed for its patterns of non-zero distinct integral solutions

Keywords: Ternary cubic ,Non- Homogeneous cubic, Integral solutions.

1. INTRODUCTION

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-13] for cubic equations with three unknowns. This communication concerns with yet another interesting equation $3x^2 + 4y^3 = z^2$ representing non-homogeneous cubic with three unknowns for determining its infinitely many non-zero integral points.

2. METHOD OF ANALYSIS

The given non-homogeneous ternary cubic diophantine equation is

$$3x^2 + 4y^3 = z^2$$

(1)

To start with, it is seen that (1) is satisfied by the integer triples given below:

$$(x, y, z) = (\pm 2\alpha^3, \alpha^2, \pm 4\alpha^3), (2n(m^2 - 3n^2), (m^2 - 3n^2), 2m(m^2 - 3n^2))$$

$$(6b(a^2 + b^2), (a^2 - 3b^2), 2a(a^2 + 9b^2)), (2s(1-3s^2), (1-3s^2), 2(1-3s^2)),$$

$$(2(k^2 \pm 2k - 2), (k^2 \pm 2k - 2), 2(k^2 \pm 2k - 2)(k \pm 1))$$

However, we have other sets of integer solutions to (1).We illustrate below
the process of obtaining different sets of integer solutions to (1):

Set 1:

Write (1) is as

$$z^2 - 3x^2 = 4y^3$$

(2)

Assume y as

$$y = a^2 - 3b^2$$

(3)

The integer 4 on the R.H.S. of (2) is expressed as

$$4 = (4 + 2\sqrt{3})(4 - 2\sqrt{3})$$

(4)

Substituting (3) and (4) in (2) and employing the method of factorization
,consider

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$$z+\sqrt{3}\,x=(4+2\sqrt{3})(a+\sqrt{3}\,b)^3$$

(5)

On equating the rational and irrational parts in (5) , one obtains

$$\left.\begin{aligned} x &= 2(a^3+9ab^2)+4(3a^2b+3b^3), \\ z &= 4(a^3+9ab^2)+6(3a^2b+3b^3) \end{aligned}\right)$$

(6)

Thus , (3) and (6) represent the integer solutions to (1).

Set 2:

Consider (1) as

$$z^2-3x^2=4y^3*1$$

(7)

The integer 1 on the R.H.S. of (7) is expressed as

$$1=(2+\sqrt{3})(2-\sqrt{3})$$

(8)

Substituting (3) ,(4) , (8) in (7) and employing the factorization method, define

$$z+\sqrt{3}\,x=2(2+\sqrt{3})^2(a+\sqrt{3}\,b)^3$$

(9)

On equating the rational and irrational parts, it is seen that

$$\left.\begin{aligned} x &= 8(a^3+9ab^2)+14(3a^2b+3b^3), \\ z &= 14(a^3+9ab^2)+24(3a^2b+3b^3) \end{aligned}\right)$$

(10)

Thus , (3) and (10) represent the integer solutions to (1).

Note :1

In addition to (8) , the integer 1 on the R.H.S. of (7) is expressed as

$$1=\frac{(3r^2+s^2+\sqrt{3}\,2rs)(3r^2+s^2-\sqrt{3}\,2rs)}{(3r^2-s^2)^2}$$

The repetition of the above process leads to a different set of solutions to (1).

Set 3:

Introduction of the transformation

$$x=z-2y$$

(11)

in (1) leads to

$$2z^2-12zy+12y^2+4y^3=0$$

Treating the above equation as quadratic in z and solving for z , one obtains

$$z=y(3\pm\sqrt{3-2y})$$

(12)

The square-root on the R.H.S. of (12) is removed when

$$y=-8k^2+12k-3$$

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(13)

and from (12) ,we have

$$z = 4k(-8k + 12k - 3), (6 - 4k)\ (-8k + 12k - 3)$$

(14)

In view of (11) , it is seen that

$$x = (4k - 2)\ (-8k + 12k - 3), (4 - 4k)\ (-8k + 12k - 3)$$

(15)

Thus,(13) ,(14) and (15) represent two sets of integer solutions to (1).

Note 2:

The square-root on the R.H.S. of (12) is also eliminated when

$$y = -8k^2 + 4k + 1$$

(16)

For this choice,the corresponding values of z and x are presented below:

$$z = (4k + 2)(-8k + 4k + 1), (4 - 4k)\ (-8k + 4k + 1)$$

(17)

$$x = 4k\ (-8k + 4k + 1), (2 - 4k)\ (-8k + 4k + 1)$$

(18)

Thus, (16) ,(17) and (18) exhibit two more sets of integer solutions to (1).

3. REMARKABLE OBSERVATION

If $(x_{0,} y_{0,} z_0)$ is any given integer solution to (1) , then ,the triple (x_n, y_n, z_n) given by

$$x_n = Y_n x_0 + X_n z_0,$$
$$z_n = 3X_n x_0 + Y_n z_0$$
$$y_n = y_0\ , n = 1,2,3,...$$

where

$$Y_n = \frac{(2+\sqrt{3})^{n+1} + (2-\sqrt{3})^{n+1}}{2},$$
$$X_n = \frac{(2+\sqrt{3})^{n+1} - (2-\sqrt{3})^{n+1}}{2\sqrt{3}}$$

satisfies (1).

4. CONCLUSION

In this paper, we have made an attempt to obtain all integer solutions to (1). To conclude, one may search for integer solutions to other choices of homogeneous or non-homogeneous ternary cubic Diophantine equations.

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