

ESTIMATING TIME OF DEATH IN FORENSIC INVESTIGATIONS USING DIFFERENTIAL EQUATIONS AND NEWTON'S LAW OF COOLING

**Dr. Jyothi. G¹, Dr. M. Dhanashmi², K. Bhanu Priya³, P. S. N. V. Sudharani⁴,
K. Dharani⁵, T. Maheswari⁶**

^{1,2,3}Lecturers, Department of Mathematics, Sri Durga Malleswara Siddhartha Mahila Kalasala, Vijayawada, A.P, India.

^{4,5,6}Students, Mathematics, Sri Durga Malleswara Siddhartha Mahila Kalasala, Vijayawada, A.P, India.

ABSTRACT

The determination of the time of death in forensic investigations is a crucial aspect of solving criminal cases. In this research paper, we propose a mathematical model based on differential equations and Newton's Law of Cooling to estimate the time of death. By measuring the body temperature at specific time intervals after the discovery of a deceased individual, we can calculate the time at which the victim's temperature was normal (98.6°F) prior to their demise. Through a detailed analysis of temperature data, our model yields a precise estimation of the time of death, aiding law enforcement agencies in their investigations.

1. INTRODUCTION

The accurate determination of the time of death is a fundamental aspect of forensic science, essential for solving criminal cases and providing justice to victims. This research endeavors to present a novel approach to estimate the time of death by leveraging mathematical modeling and the principles of heat transfer. Our methodology is based on Newton's Law of Cooling, which describes how an object loses heat to its surroundings over time. In this context, the "object" is the deceased body, and the "surroundings" is the room's temperature, which remains constant. By formulating a differential equation that relates the body's temperature to time, we can estimate when the victim's temperature was at a normal level of 98.6°F before death. Blanchard, Devaney, and Hall (2011) present a comprehensive overview of differential equations in their textbook "Differential Equations." This work provides a strong foundation in the theoretical aspects of differential equations, covering topics from basic concepts to advanced techniques.

It serves as an essential resource for students and researchers seeking a deep understanding of the subject. Tenenbaum and Pollard's classic text "Ordinary Differential Equations" (1985) has been a cornerstone reference in the study of ordinary differential equations. This work delves into both theory and practical applications, offering valuable insights into solving differential equations in various contexts. Nagle, Saff, and Snider's "Fundamentals of Differential Equations" (2017) is a contemporary textbook that bridges theory and application. It provides a modern perspective on solving differential equations and incorporates practical examples from engineering, physics, and other disciplines, making it suitable for a wide range of learners. E. L. Ince's "Ordinary Differential Equations" (1926) is a seminal work that laid the foundation for the study of ordinary differential equations. While it may be considered a historical reference, Ince's contributions to the field are enduring, and his text remains a valuable resource for understanding classical approaches to differential equations. V. I. Arnold's "Ordinary Differential Equations" (1992) is a highly regarded text that offers a more advanced perspective on the subject. Arnold's work delves into the theory of differential equations, providing insights into their dynamical systems aspects. It is an essential resource for researchers and mathematicians interested in the deeper mathematical theory of differential equations. Simmons' **Differential Equations:**

"Theory, Technique, and Practice" (2016) provides a modern and comprehensive approach to the study of differential equations. This text combines theoretical foundations with practical techniques, making it suitable for students and researchers alike.

It reflects contemporary developments in the field and offers insights into recent advancements in solving differential equations. Differential equations continue to be a vibrant and evolving field of mathematics, with applications spanning numerous scientific and engineering disciplines. The works referenced in this literature review encompass a wide range of topics, from foundational theory to advanced applications, reflecting the enduring importance and relevance of differential equations in contemporary mathematics and scientific research. Whether as foundational textbooks for students or as references for researchers, these texts play a crucial role in advancing our understanding and application of differential equations.

2. MATHEMATICAL MODELING OF THE PROBLEM

The estimation of the time of death in a homicide case can be achieved through mathematical modeling using differential equations. When a police officer finds the body of a deceased individual, the objective is to approximate the moment of death.

This investigation takes place in an environment where the room temperature is consistently maintained at 70 degrees Fahrenheit. Now that the officer has the temperature function describing the body's cooling process, the next step is to pinpoint when the body's temperature reached the crucial 98.6°F, which is presumed to be the time of death. This requires solving the equation for time. Forensic expert will try to estimate this time from body's current temperature and calculating how long it would have had to lose heat to reach this point.

According to Newton's law of cooling, the body will radiate heat energy into the room at a rate proportional to the difference in temperature between the body and the room. If $T(t)$ is the body temperature at time t , then for some constant of proportionality k ,

$$T'(t) = k[T(t) - 70]$$

This is a separable differential equation and is written as

$$\frac{1}{T - 70} dT = k dt$$

Upon integrating both sides, one gets

$$\ln|T - 70| = kt + c$$

Taking exponential, one gets

$$|T - 70| = e^{kt+c} = Ae^{kt}$$

where $A = e^c$. Then

$$T - 70 = \pm Ae^{kt} = Be^{kt}$$

Then

$$T(t) = 70 + Be^{kt}$$

The values of constants k and B , which are crucial for our estimation process, can be ascertained when we have access to specific information. This information includes the time at which the police personnel arrived, the initial temperature of the body right after their arrival, and the subsequent body temperature readings. Let the officer arrived at 10.40 p.m. and the body temperature was 94.4 degrees. This means that if the officer considers 10:40 p.m. as $t=0$ then

$$T(0) = 94.4 = 70 + B \text{ and so}$$

$$B = 24.4 \text{ giving}$$

$$T(t) = 70 + 24.4 e^{kt}$$

Let the officer makes another measurement of the temperature say after 90 minutes, that is, at 12.10 a.m. and temperature was 89 degrees. This means that

$$T(90) = 89 = 70 + 24.4 e^{90k}$$

Then

$$e^{90k} = \frac{19}{24.4},$$

so

$$90k = \ln\left(\frac{19}{24.4}\right)$$

and

$$k = \frac{1}{90} \ln\left(\frac{19}{24.4}\right)$$

The officer has now temperature function

$$T(t) = 70 + 24.4 e^{\frac{t}{90} \ln\left(\frac{19}{24.4}\right)}$$

In order to find when the last time the body was 98.6 (presumably the time of death), one has to solve for time the equation

$$T(t) = 98.6 = 70 + 24.4 e^{\frac{t}{90} \ln\left(\frac{19}{24.4}\right)}$$

To do this, the officer writes

$$\frac{28.6}{24.4} = e^{\frac{t}{90} \ln\left(\frac{19}{24.4}\right)}$$

and applying logarithms on both sides to obtain

$$\ln\left(\frac{28.6}{24.4}\right) = \frac{t}{90} \ln\left(\frac{19}{24.4}\right)$$

Therefore, the time of death, according to this mathematical model, was

$$t = \frac{90 \ln(28.6/24.4)}{\ln(19/24.4)} \text{ which is approximately } -57.07 \text{ minutes.}$$

From this equation, the time of death can be deduced. In this mathematical model, the calculated time of death is approximately -57.07 minutes. This suggests that the death occurred approximately 57.07 minutes prior to the first temperature measurement at 10:40 p.m., which would place it around 9:43 p.m. approximately.

3. RESULTS AND CONCLUSION

In our research, we applied this methodology to a specific case where the initial temperature was 94.4°F at 10:40 p.m. and 89°F after 90 minutes. Through rigorous mathematical calculations, we estimated the time of death to be approximately 9:43 p.m., which was 57.07 minutes before the initial measurement. This approach provides a powerful tool for forensic experts and law enforcement agencies to estimate the time of death with greater accuracy, aiding in criminal investigations. By combining mathematical modeling with real-world temperature measurements, our method offers a promising avenue for advancing forensic science and improving the resolution of criminal cases. Further research and validation of this approach are warranted to enhance its reliability and applicability in various forensic scenarios.

4. REFERENCES

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