INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)
(Int Peer Reviewed Journal)
Vol. 04, Issue 09, September 2024, pp : 1116-1131
www.ijprems.com
editor@ijprems.com
e-ISSN : 2583-1062
Impact Factor : 7.001

FLEXURAL AND SPLIT TENSILE STRENGTHS INVESTIGATION OF STEEL FIBRE REINFORCED CONCRETE [SFRC] USING SCHEFFE' S (5,2) OPTIMIZATION MODEL

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ABSTRACT

One of the fastest ways to obtain desiring mix proportion for a concrete without going through rigorous procedures is by optimization. Through optimization, the advantages of investigating the SFRC mechanical properties such as Flexural Strength and Split Tensile Strength can be maximised. This research work is therefore aimed at using Scheffe's Second Degree Model for five component mixture to optimize the Flexural Strength and Split Tensile Strength of Steel Fibre Reinforced Concrete [SFRC]. Using Scheffe's Simplex method, the Flexural Strength and Split Tensile Strength of SFRC were determined for different fifteen mix proportions. Fifteen control experiments were also carried out and the flexural and split tensile strengths determined. The result of the student's t-test shows that the strengths predicted by the model and the corresponding experimentally observed results are highly correlated. Maximum design strengths recorded for the flexural test at 14 and 28 days were 6.24MPa and 7.06MPa respectively, while those recorded for the splitting tensile test were 3.53MPa and 5.42MPa respectively. Thus, considering its safety and economic advantages SFRC controllable design strength values can easily find applications as concrete flooring for parking lots, playgrounds, airport runways, taxiways, maintenance hangars, access roads, workshops, port pavements, container storage and handling areas, bulk storage warehouses, and military warehouses.

Keywords: Flexural Strength, Split Tensile Strength , Scheffe's (5,2) Optimisation Model, SFRC, Mixture Design

1. INTRODUCTION

Optimization of Steel Fibre Reinforced Concrete (SFRC) mixture design can be described as a process of search for a mixture for which the sum of the costs of the ingredients making up the SFRC mixture (that is, water, cement, fine aggregate, coarse aggregate and steel fibre) is lowest, yet satisfying the major required performance of concrete, such as workability, flexibility, homogeneity, strength, durability etc. Generally, an optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, or constraints placed on the variables (which might be time, machine or labour) concerned. The objective of mix design, according to Shacklock (1974), is to determine the most appropriate mixes in which to use the component materials to meet the needs of construction work. According to Jackson and Dhir (1996), concrete mix design remains the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. From the above definition, the cost of any concrete should include, in addition to that of the materials themselves, the cost of the mix design, batching, mixing and placing the concrete and the site supervision. Going by the above guidelines, the empirical mix design methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) are a little bit complex as well as time consuming as they involve a lot of trial mixes and complex statistical computations before the desired strength of the concrete can be obtained. Thus, optimization of the concrete mixture design is more preferable, as it remains the fastest method, more result oriented, best option, most convenient and the most efficient way of selecting concrete mix ratios for better efficiency and better performance of concrete. A typical example of optimization model is Scheffe's Model which could be in the form of Scheffe's Second Degree Model or Scheffe's Third Degree Model. In this present study, Scheffe's Second Degree Model for five components mixtures (namely Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Steel Fibre) is presented.

It is an established fact that concrete is one of the most widely used materials in the construction industries, second only to water. It has also been undergoing changes both as a material and due to advancement in technology and researches. .According to Neville (1990), concrete plays an important part in all building and civil engineering structures owing to its numerous advantages which ranges from low built in fire resistance, high compressive strength to low maintenance, etc. By definition, concrete is a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water (Oyenuga, 2008). Again, according to Syal and Goel (2007), concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as construction materials throughout the world. One of the drawbacks that concrete has is that it is strong in compression and weak in tension. To remedy this situation in an unreinforced concrete, conventional steel reinforcement are usually provided to address the weak portion of the concrete. But the conventional reinforcements are expensive and many household may find it very difficult to cope with the associated increased cost of concrete production. One of the research output has seen the expensive conventional reinforced bars being substituted by less expensive fibres. This type of research is known as Fibre reinforced concrete (FRC) research. In general, FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete as well as uniformly dispersed fibre. The main purposes of incorporating the fibrous materials remain to increase the concrete's durability and structural integrity and at the same time save costs. Steel Fibre Reinforced Concrete (SFRC) is one form of FRC and is the concrete mixture where the conventionally steel reinforcement in concrete production is replaced with steel fibre. It is worthy to note that in a typical reinforced concrete member, the knowledge of tensile and split tensile strengths of concrete is also of utmost importance to the concrete designer. It is also important to note that one of the ultimate aims of studying the various properties of the materials of concrete is to enable concrete technologist to design a concrete mix for a particular strength and durability (Shetty, 2006). Thus, for this present work, special properties of SFRC under investigation are the flexural strength and the split tensile strength. By definition, flexural strength (always described as modulus of rupture) is the ability of the material to withstand bending forces applied perpendicular to its longitudinal axis. It is also defined as the maximum bending stress that can be applied to the material before it yields. On the other hand, splitting tensile strength test on concrete cylinder is a method to determine the tensile strength of concrete. It is generally carried out to obtain the tensile strength of concrete, and the stress field in the tests is actually a biaxial stress field with compressive stress three times greater than the tensile stress. The split tensile strength test is an indirect method of testing tensile strength of concrete and is generally greater than direct tensile strength and lower than flexural strength.

This present study examines the application of Scheffe's Second Degree Model for five component mixture in the optimization of the Flexural Strength and Split Tensile Strength of SFRC. Although, some related works have been done by many researchers, none has been able to address the core subject matter. For instance, on SFRC and related works, Baros and others (2005) investigated the post – cracking behaviour of SFRC. Jean-Louis and Sana (2005) investigated the corrosion of SFRC from the crack. Lima and Oh (1999) carried out an experimental and theoretical investigation on the shear of SFRC beams. Similarly, Lau and Anson (2006) carried out research on the effect of high temperatures on high performance SFRC. The work of Lie and Kodar (1996) was on the study of thermal and mechanical properties of SFRC at elevated temperatures. Blaszczynski and Przybylska-Falek (2015) investigated the use of SFRC as a structural material. Huang and Zhao (1995) investigated the properties of SFRC containing larger coarse aggregate. Arube and others (2021) investigated the Effects of Steel Fibres in Concrete Paving Blocks. Again, Khaloo and others (2005) examined the flexural behaviour of small SFRC slabs. And Ghaffer and others (2014) investigated the use of steel fibres in structural concrete to enhance the mechanical properties of concrete. Recent works on optimization show that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For instance, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were based on the use of Scheffe' model in the optimization of compressive strength of Perwinkle Shell-Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Mbadike and Osadebe (2013) applied Scheffe's (4,2) model to optimize the compressive strength of Laterite Concrete. Egamana and Sule (2017) carried out an optimization work on the compressive strength of periwinkle shell aggregate concrete Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3) where comparison was made between second degree model and third degree model. Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied

Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation ,Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid- Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC) .Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid-Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2023c) applied Scheffe's (6,2) model to optimize the Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC). Finally, Nwachukwu and others (2023d) made use of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC). Nwachukwu and others (2023e) used optimization techniques in the Flexural Strength And Split Tensile Strength determination of Hybrid Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023f) applied Scheffe's Optimization model in the evaluation of Flexural Strength And Split Tensile Strength Of Plastic Fibre Reinforced Concrete (PLFRC). Nwachukwu and Opara (2023) in their paper presented at the Conference Proceedings of the Nigeria Society of Engineers, demonstrated the use of Snail Shells Ash (SSA) in the partial replacement of cement using Scheffe's (5,2) optimization model. Nwachukwu and others (2024a) applied the use of Scheffe's (6,2) model to evaluate the optimum flexural and split tensile strengths of Periwinkle Shells Ash (PSA)- Mussel Shells Ash (MSA)- Cement Concrete (PMCC). Nwachukwu and others (2024b) applied the use of Scheffe's (6,2) model to evaluate the optimum compressive strength of Periwinkle Shells Ash (PSA)- Snail Shells Ash (SSA)- Cement Concrete (PSCC). Nwachukwu and others (2024c) applied Scheffe's (5,2) model to evaluate the compressive strength of Plastic Fibre Reinforced Concrete [PLFRC]**.** Nwachukwu and others (2024d) applied the use of Scheffe's Third Degree Model to optimize the compressive strength of HPNFRC. Nwachukwu and others (2024e) applied the use of Scheffe's Third Degree Regression Model to optimize the compressive strength of MSFRC. Nwachukwu and others (2024f) applied the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of NFRC. Again, Nwachukwu and others (2024g) applied the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of PFRC. Finally, Nwachukwu and others (2024h) applied the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of PFRC. Evidences from the works reviewed so far show that no work has been done on the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of SFRC. Thus, there is urgent need for this present research work.

2. METHODOLOGY

2.1 MATERIALS FOR SFRC-FSTS MIXTURES

For this present SFRC work, the component materials under Flexural And Split Tensile Strengths [FSTS] investigation in line with Scheffe's (5,2) model are Water/Cement ratio, Cement, Fine and Coarse Aggregates and Steel Fibre. Potable water is obtained from clean water source and was used in accordance with ASTM C1602/C1602M-22 (2022). The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). Fine aggregate, within 0.05 - 4.5mm sizes was procured from the local river. Crushed granite (as a coarse aggregate) of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. Both fine and coarse aggregates are procured and prepared in accordance with ASTM C33/C33M-18 (2018). The same size and nature of steel fibre used previously by Nwachukwu and others (2022b) and Nwachukwu and others (2022g) in the compressive strength investigation, is the same as the one being used in this present work based on Scheffe's second degree model.

2.2. BASIC THEORITICAL INFORMATION ON SFRC SCHEFFE'S (5, 2) MODEL

By definition, a simplex lattice is a structural representation of lines joining the atoms of a mixture. it is important to know that these atoms are the constituent components of the same mixture. For instance, when considering this present SFRC concrete mixture, the five constituent elements are Water, Cement, Fine Aggregate, Coarse Aggregate and Steel Fibre. One basic information to know, according to Obam (2009) is that the mixture components are usually subject to the constraint that the sum of all the components must be equal to 1. This is stated mathematically in Eqn. (1): $X_1 +$

$X_2 + X_3 + ... + X_q = 1$; $\Rightarrow \sum_{i=1}^{q} X_i = 1$ **(1)** where $X_i \geq 0$ and i = 1, 2, 3… q, and q = the number of mixtures.

2.2.1. POSSIBLE DESIGN POINTS FOR SFRC SCHEFFE'S (5, 2) COMPONENT MIXTURES

The (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains $^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, ..., 1$; $i = 1, 2, ..., q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degree, which in this present study is 2.

For example a (3, 2) lattice consists of $^{3+2-1}C_2$ i.e. $^4C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1, 0, 0), (0, 1, 0), (0, 0, 1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. In order to determine the number of coefficients/terms/ design points required for a given Scheffe's component mixtures, the following general formula is applied: $k = \frac{(q+m-1)!}{(q-1)!\,.\,m!}$ Or $^{q+m-1}C_m$ **2(a-b)**

Where k = number of coefficients/ terms / design points , q = number of components = 5 in this work and m = number of degree of polynomial = 2 in this present work. .Using either of Eqn. (2), $k_{(5,2)} = 15$.

Consequently, the possible Pseudo design points for SFRC Scheffe's (5,2) lattice can be as follows:

A_1 (1,0,0,0,0,); A_2 (0,1,0,0,0,); A_3 (0,0,1,0,0,); A_4 (0,0,0,1,0,), A_5 (0,0,0,0,1 A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0,0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) and A_{45} (0, 0, 0, 0.5, 0.5) **(3)**

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3, X_4 … X_q is given in the form of Eqn.(4): $P = b_0 + \sum bi\, xi + \sum bij\mathcal{x}j + \sum bi\, j\mathcal{x}j\mathcal{x}k + \sum bi\,^{j}{}_{2} + ...\, i_n\mathcal{x}i_2\mathcal{x}i_n$ **(4)**

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq ... \leq i_n \leq q$ respectively) , b = constant coefficients and P is the response (the response is a polynomial function of pseudo component of the mix) which represents the property under study, which ,in this case is the Flexural Strength (P^F) or Split Tensile Strength (P^S) as the case may be. As this research work is based on the Scheffe's (5, 2) simplex, the actual form of Eqn. (4) has already been developed by Nwachukwu and others (2017) and will be applied subsequently

2.2.2. PSEUDO AND ACTUAL COMPONENTS IN SFRC SCHEFFE'S MIXTURE

In a typical Scheffe's mixture design, the relationship between the pseudo components and the actual components is stated as: $Z = A * X$ **(5)**

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship.

When we re-arrange Eqn. (5), we have: $X = A^{-1} * Z$ **(6)**

2.2.3. FORMULATION OF MATHEMATICAL EQUATION FOR SFRC – FSTS SCHEFFE'S (5, 2) SIMPLEX LATTICE

The polynomial equation by Scheffe (1958), describing the response is stated in Eqn.(4). But, for Scheffe's (5,2) simplex lattice, the polynomial equation for five component mixtures has been derived from Eqn.(4) by Nwachukwu and others (2017) and the simplified version is given as follows:

$P = ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 + ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{45}X_4X_5$ **(7)**

2.2.4. COEFFICIENTS DETERMINATION OF THE SFRC SCHEFFE'S (5,2) POLYNOMIAL

From the work of Nwachukwu and others (2022h), the simplified equations for the coefficients of the Scheffe's (5, 2) polynomial are expressed as follows. :

$\beta_1 = P_1$; $\beta_2 = P_2$; $\beta_3 = P_3$; $\beta_4 = P_4$; $\beta_5 = P_5$; $\beta_{12} = 4P_{12} - 2P_1 - 2P_2$; $\beta_{13} = 4P_{13} - 2P_1 - 2P_3$; **8(a-g)**

$\beta_{14} = 4P_{14} - 2P_1 - 2P_4$; $\beta_{15} = 4P_{15} - 2P_1 - 2P_5$; $\beta_{23} = 4P_{23} - 2P_2 - 2P_3$; $\beta_{24} = 4P_{24} - 2P_2 - 2P_4$; **9(a-d)**

$\beta_{25} = 4P_{25} - 2P_2 - 2P_5$; $\beta_{34} = 4P_{34} - 2P_3 - 2P_4$; $\beta_{35} = 4P_{35} - 2P_3 - 2P_5$; $\beta_{45} = 4P_{45} - 2P_4 - 2P_5$ **10(a-d)** Where P_i = Response Function (Flexural Strength or Split Tensile Strength) for the pure component, i

2.2.5. SCHEFFE'S (5, 2) MIXTURE DESIGN MODEL FOR SFRC

When we substitute Eqns. (8)-(10) into Eqn. (7), we obtain the mixture design model for the SFRC mixture based on Scheffe's (5, 2) lattice for the flexural and split tensile strengths.

2.2.6. ACTUAL AND PSEUDO MIX RATIOS FOR THE SFRC- FSTS SCHEFFE'S (5,2) DESIGN LATTICE AT INITIAL EXPERIMENTAL TEST POINT[IETP] AND EXPERIMENTAL CONTROL TEST POINT[ECTP]

A. AT THE SFRC-FSTS IETP

From the practical knowledge point of view, the requirement of simplex lattice design from Eqn.(1) makes it impossible to use the conventional mix ratios such as 1:2:4, or 1:3:6, as the case may be., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions can be chosen for the five points vertices:

A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8);A_3 (0.5:1:1.2:1.7:1);A_4 (0.7:1:1:1.8:1.2)

and A_5 (0.75:1:1.3:1.2:1.5) **(11)**

From Eqn.(11), the mix ratios represent water/cement ratio, cement, fine aggregate, coarse aggregate and steel fibre. For the pseudo mix ratio, we have the following corresponding mix ratios at the vertices from Eqn.(3):

A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0) and A_5(0:0:0:0:1) **(12)**

For the transformation of the actual component, Z to pseudo component, X, and vice versa , Eqns.(5)and (6) are used. Substituting the mix ratios from point A_1 into Eqn. (5) we have:

$$\begin{Bmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{pmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad \textbf{(13)}$$

Transforming the R.H matrix and solving , we obtain:

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} \quad \textbf{(14)}$$

Thus,

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} = \begin{pmatrix} 3.99 & 10.37 & -2.14 & -3.05 & -4.62 \\ -4.88 & -21.46 & 5.40 & 5.95 & 7.31 \\ -1.78 & 17.83 & -3.49 & -4.20 & -4.62 \\ 1.04 & -9.24 & 0.37 & 3.28 & 2.69 \\ 1.63 & 3.49 & -0.13 & -1.98 & -0.77 \end{pmatrix} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} \quad \textbf{(15)}$$

Considering the mix ratios at the midpoints of Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn. (15) , we obtain the corresponding actual mix ratio as shown under: For example, we have at point A_{12}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.62 \\ 1 \\ 1.65 \\ 1.90 \\ 0.65 \end{Bmatrix} \quad \textbf{(16)}$$

Solving; $Z_1 = 0.62$, $Z_2 = 1$, $Z_3 = 1.65$, $Z_4 = 1.9$, $Z_5 = 0.65$ and the rest are shown in Table 1

To generate the fifteen polynomial coefficients, fifteen experimental tests (each for Flexural Strength and Split Tensile Strength) will be carried out and the corresponding mix ratio is as depicted in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio For SFRC- FSTS Based On Scheffe's (5,2) Lattice For IETP (For Flexural Strength And Split Tensile Strength).

S/N	IETP	PSEUDO COMPONENT					RESPONSE SYMBOL	ACTUAL COMPONENT				
		X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	E_I	1	0	0	0	0	P_1	0.67	1	1.70	2.00	0.50
2	E_2	0	1	0	0	0	P_2	0.56	1	1.60	1.80	0.80
3	E_3	0	0	1	0	0	P_3	0.50	1	1.20	1.70	1.00
4	E_4	0	0	0	1	0	P_4	0.70	1	1-00	1.80	1.20
5	E_5	0	0	0	0	1	P_5	0.75	1	1.30	1.20	1.50
6	E_{12}	0.50	0.50	0	0	0	P_{12}	0.62	1	1.65	1.90	0.65
7	E_{13}	0.50	0	0.50	0	0	P_{13}	0.59	1	1.45	1.85	0.75
8	E_{I4}	0.50	0	0	0.50	0	P_{14}	0.69	1	1.35	1.90	0.85
9	E_{I5}	0.50	0	0	0	0.50	P_{15}	0.71	1	1.50	1.60	1.00
10	E_{23}	0	0.50	0.50	0	0	P_{23}	0.53	1	1.40	1.75	0.90
11	E_{24}	0	0.50	0	0.50	0	P_{24}	0.63	1	1.30	1.80	1.00
12	E_{25}	0	0.50	0	0	0.50	P_{25}	0.66	1	1.45	1.50	1.15
13	E_{34}	0	0	0.50	0.50	0	P_{34}	0.60	1	1.10	1.75	1.10
14	E_{35}	0	0	0.50	0	0.5	P_{35}	0.63	1	1.25	1.45	1.25
15	E_{45}	0	0	0	0.5	0.5	P_{45}	0.73	1	1.15	1.50	1.50

B. AT THE SFRC-FSTS ECTP

For the purpose of this research, fifteen different controls test (each for Flexural Strength and Split Tensile Strength) were predicted which according to Scheffes, their summation should not be more than one. Thus, the following pseudo mix proportions (whose summation should not be more than one) are applicable at the control points:

C_1 (0.25, 0.25, 0.25, 0.25, 0), C_2 (0.25, 0.25, 0.25, 0, 0.25), C_3 (0.25, 0.25, 0, 0.25, 0.25), C_4 (0.25, 0, 0.25, 0.25, 0.25), C_5 (0, 0.25, 0.25, 0.25, 0.25), C_{12} (0.20, 0.20, 0.20, 0.20, 0.20), C_{13} (0.30, 0.30, 0.30, 0.10, 0), C_{14} (0.30, 0.30, 0.30, 0, 0.10), C_{15} (0.30, 0.30, 0, 0.30, 0.1), C_{23} (0.30, 0, 0.30, 0.30, 0.1), C_{24} (0, 0.30, 0.30, 0.30, 0.10), C_{25} (0.10, 0.30, 0.30, 0.30, 0), C_{34} (0.30, 0.10, 0.30, 0.30, 0), C_{35} (0.30, 0.30, 0.10, 0.30, 0), C_{45} (0.10, 0.20, 0.30, 0.40, 0), **(17)**

Substituting into Eqn.(16) , we obtain the values of the actual mixes as follows:

At Control 1, C_1

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix} = \begin{Bmatrix} 0.61 \\ 1 \\ 1.38 \\ 1.8 \\ 0.5 \end{Bmatrix} \quad (18)$$

The rest are depicted in Table 2

Table 2: Actual (Z_i) and Pseudo (X_i) Component of SFRC Scheffe's (5, 2) Simplex Lattice At ECTP (For Flexural Strength And Split Tensile Strength).

S/N	RESPONSE SYMBOL	PSEUDO COMPONENTS					EC TP	ACTUAL COMPONENTS				
		X_1	X_2	X_3	X_4	X_5		Z_1	Z_2	Z_3	Z_4	Z_5
1	P_1	0.25	0.25	0.25	0.25	0.00	C_1	0.61	1	1.38	1.83	0.50
2	P_2	0.25	0.25	0.25	0.00	0.25	C_2	0.62	1	1.45	1.68	0.80
3	P_3	0.25	0.25	0.00	0.25	0.25	C_3	0.67	1	1.40	1.70	1.00
4	P_4	0.25	0.00	0.25	0.25	0.25	C_4	0.66	1	1.30	1.68	1.20

5	P_5	0.00	0.25	0.25	0.25	0.25	C_5	0.63	1	1.28	1.63	1.50
6	P_{12}	0.20	0.20	0.20	0.20	0.20	C_{12}	0.64	1	1.36	1.70	0.65
7	P_{13}	0.30	0.30	0.30	0.10	0.00	C_{13}	0.59	1	1.45	1.83	0.75
8	P_{14}	0.30	0.30	0.30	0.00	0.10	C_{14}	0.59	1	1.48	1.77	0.85
9	P_{15}	0.30	0.30	0.00	0.30	0.10	C_{15}	0.65	1	1.42	1.80	1.00
10	P_{23}	0.30	0.00	0.30	0.30	0.10	C_{23}	0.64	1	1.30	1.77	0.90
11	P_{24}	0.00	0.30	0.30	0.30	0.10	C_{24}	0.60	1	1.27	1.71	1.00
12	P_{25}	0.10	0.30	0.30	0.30	0.00	C_{25}	0.60	1	1.31	1.79	1.15
13	P_{34}	0.30	0.10	0.30	0.30	0.00	C_{34}	0.62	1	1.33	1.83	1.10
14	P_{35}	0.30	0.30	0.10	0.30	0.00	C_{35}	0.63	1	1.41	1.85	1.25
15	P_{45}	0.10	0.20	0.30	0.40	0.00	C_{45}	0.61	1	1.25	1.79	0.50

2.2.7. MEASUREMENT OF QUANTITIES OF SFRC- FSTS MATERIALS

The actual component as transformed from Eqn. (14), Tables (1) and (2) were used to measure out the quantities of Water/Cement Ratio (Z_1), Cement (Z_2), Fine Aggregate (Z_3), Coarse Aggregate (Z_4), and Steel Fibre (Z_5) using a weighing balance of 50kg capacity in their respective ratios for the eventual Concrete Beam Cube and Concrete Cylindrical specimen at the laboratory.

Mathematically, Measured Quantity, M^Q of SFRC Mixture is given by Eqn.(19)

$$M^Q = \frac{X}{T} * Y \quad \textbf{(19)}$$

Where, X = Individual mix ratio at each test point. For example, X = 0.67 for Z_1 at E_1 in Table 1.

T = Sum of mix ratios at each test point = 5.87 at E_1 in Table 1.

And Y = Average weight of Concrete cube/beam/cylinder

For the Flexural Strength concrete beam mould of 15cm*15cm*60cm, Average Y from experience = 30kg

For the Split Tensile Strength Concrete cylinder mould of 15cm*30cm, Average Y from experience =12.5kg

Samples of measured quantities can be seen from the works of Nwachukwu and others 2024 (a and b).

2.3. METHOD

2.3.1. METHODS FOR SFRC FLEXURAL STRENGTH TEST

A. SFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR FLEXURAL STRENGTH TEST

In this experimental investigation, the standard size of specimen (mould) for the Flexural Strength measures 15cm*15cm*60cm. The mould is made of steel metal with sufficient thickness to prevent spreading or warping. The mould is constructed with the longer dimension horizontal and in such a manner as to facilitate the removal of the moulded specimen without damage. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cubes. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (10). Twenty-four (24) hours after moulding, curing commenced. Test specimens are stored in water at a temperature of 24^0 to 30^0 for 48 hours before testing. They are tested immediately on removal from the water whilst they are still in a wet condition. After 14 days and 28 days of curing respectively, the specimens were taken out of the curing tank for flexural strength determination.

B. SFRC FLEXURAL STRENGTH TEST PROCEDURE/CALCULATION

Flexural strength testing was done in accordance with BS 1881 – part 118 (1983) - Method of determination of Flexural Strength, ASTM C78/C78M-22 (2022) and ACI (1989) guideline. In this present study, two samples were crushed for each mix ratio. In each case, the Flexural Strength of each sample which is expressed as the Modulus of Rupture (MOR) was then calculated to the nearest 0.05 MPa using Eqn.(20)

$$MOR = \frac{PL}{bd^2} \quad \textbf{(20)}$$

where b = measured width in cm of the specimen, d = measured depth in cm of the specimen at the point of failure, where L = Length in cm of the span on which the specimen was supported and P = maximum load in kg applied to the specimen.

2.3.2. METHODS FOR SFRC SPLIT TENSILE STRENGTH TEST

A. SFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR SPLIT TENSILE STRENGTH TEST

The specimen for the Split Tensile Strength is Concrete Cylindrical specimen measuring diameter 150 mm and length 300 mm. They were cast with plastic fibers and the specimen was loaded for ultimate compressive load under Universal Testing Machine (UTM) for each mix. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cylinders. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (10).. After 14days and 28 days of curing the specimens were taken out of the curing tank for the Split Tensile Strength determination.

B. SFRC SPLIT TENSILE STRENGTH TEST PROCEDURE/CALCULATION

The cylindrical split tensile test was done using the universal testing machine in accordance with BS EN 12390-6:2009 and ASTM C 496/ C 496 M-17 (2017). Two samples were crushed for each mix ratio and each case, the Split Tensile Strength of each specimen/sample was then calculated using Eqn. (21) .

$$F_t = \frac{2P}{\pi D L} \quad \textbf{(21)}$$

Where, F_t = Split Tensile Strength, MPa , P = maximum applied load (that is Load at failure, N) ; D = diameter of the cylindrical specimen (Dia. Of cylinder, mm); and L = Length of the specimen (Length of cylinder, mm),

3. RESULTS PRESENTATION AND DISCUSSION

3.1. SFRC RESPONSES (FLEXURAL STRENGTH) FOR THE IETP.

The results of the Flexural Strength (responses) test based on Eqn. (20) are shown in Table 3

Table 3: SFRC Flexural Strength (Response) For IETP Based on Eqn.(20)

S/N	IETP	EXPERIMENTAL NO.	RESPONSE SYMBOL	RESPONSE P_i, MPa		$\sum P_i$		AVERAGE RESPONSE P, MPa	
				14th day Results	28th day Results	14th day Results	28th day Results	14th day Results	28th day Results
1	E_I	SFRC/FS/E_1 A SFRC/FS/ E_1 B	P_1	4.00 4.08	4.28 4.23	8.08	8.51	4.04	4.26
2	E_2	SFRC/FS/ E_2 A SFRC/FS/ E_2 B	P_2	3.96 3.84	4.02 3.98	7.80	8.00	3.90	4.00
3	E_3	SFRC/FS/ E_3 A SFRC/FS/ E_3 B	P_3	5.23 5.27	6.08 6.10	10.50	12.18	5.25	6.09
4	E_4	SFRC/ FS/E_4 A SFRC/FS/ E_4 B	P_4	5.00 5.10	6.23 6.29	10.10	12.52	5.05	6.26
5	E_5	SFRC/FS/ E_5 A SFRC/FS/ E_5 B	P_5	5.22 5.28	5.14 5.22	10.50	10.36	5.25	5.18
6	E_{12}	SFRC/FS/ E_{12} A SFRC/FS/ E_{12} B	P_{12}	5.21 5.19	5.46 5.48	10.40	10.94	5.20	5.47
7	E_{13}	SFRC/FS/ E_{13} A SFRC/FS/ E_{13} B	P_{13}	4.98 4.92	5.28 5.36	9.90	10.64	4.95	5.32
8	E_{I4}	SFRC/FS/ E_{14} A SFRC/FS/ E_{14} B	P_{14}	4.25 4.29	4.28 4.30	8.58	8.58	4.27	4.29
9	E_{I5}	SFRC/FS/ E_{15} A SFRC/FS/ E_{15} B	P_{15}	4.30 4.32	4.86 4.90	8.62	9.76	4.31	4.88
10	E_{23}	SFRC/ FS/E_{23} A SFRC/FS/ E_{23} B	P_{23}	3.98 4.12	4.81 4.92	8.10	9.73	4.05	4.87
11	E_{24}	SFRC/FS/ E_{24} A		6.26	7.08	12.48	14.12	6.24	7.06

INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)
(Int Peer Reviewed Journal)
Vol. 04, Issue 09, September 2024, pp : 1116-1131
www.ijprems.com
editor@ijprems.com
e-ISSN : 2583-1062
Impact Factor : 7.001

		SFRC/FS/ E_{24} B	P_{24}	6.22	7.04				
12	E_{25}	SFRC/FS/ E_{25} A SFRC/FS/ E_{25} B	P_{25}	5.26 5.32	6.98 6.94	10.58	13.92	5.29	6.96
13	E_{34}	SFRC/FS/ E_{34} A SFRC/FS/ E_{34} B	P_{34}	4.18 4.21	6.82 6.88	8.39	13.70	4.20	6.85
14	E_{35}	SFRC/ FS/E_{35} A SFRC/FS/ E_{35} B	P_{35}	4.36 4.27	6.12 6.22	8.63	12.34	4.32	6.17
15	E_{45}	SFRC/FS/ E_{45} A SFRC/FS/ E_{45} B	P_{45}	6.12 6.23	5.92 5.94	12.35	11.86	6.18	5.93

3.2 SFRC RESPONSES (SPLIT TENSILE STRENGTH) FOR THE IETP

The results of the Split Tensile Strength (response) for the IETP based on Eqn. (21) are shown in Table 4

Table 4: SFRC Split Tensile Strength (Response) For The IETP Based on Eqn.(21)

S/N	IETP	EXPERIMENTAL NO	RESPONSE SYMBOL	RESPONSE P_i, MPa		$\sum P_i$		AVERAGE RESPONSE,P, MPa	
				14th day Results	28th day Results	14th day Results	28th day Results	14th day Results	28th day Results
1	E_I	SFRC/STS/E_1 A SFRC/STS/E_1 B	P_1	2.22 2.28	3.60 3.64	4.50	7.24	2.25	3.62
2	E_2	SFRC/STS/ E_2 A SFRC/STS/ E_2 B	P_2	2.10 2.10	3.56 3.46	4.20	7.02	2.10	3.51
3	E_3	SFRC/STS/ E_3 A SFRC/STS/ E_3 B	P_3	2.99 3.10	3.75 3.77	6.09	7.52	3..05	3.76
4	E_4	SFRC/STS/E_4 A SFRC/STS/E_4 B	P_4	3.54 3.58	3.82 3.87	7.12	7.69	3.56	3.85
5	E_5	SFRC/STS/ E_5 A SFRC/STS/ E_5 B	P_5	2.84 2.88	4.22 4.28	5.72	8.50	2.86	4.25
6	E_{12}	SFRC/STS/ E_{12} A SFRC/STS/ E_{12} B	P_{12}	2.45 2.50	4.85 4.87	4.95	9.72	2.48	4.86
7	E_{13}	SFRC/STS/ E_{13} A SFRC/STS/ E_{13} B	P_{13}	3.10 3.20	4.95 5.00	6.30	9.95	3.15	4.98
8	E_{I4}	SFRC/STS/E_{14} A SFRC/STS/ E_{14} B	P_{14}	3.28 3.22	4.27 4.28	6.50	8.55	3.25	4.28
9	E_{I5}	SFRC/STS/ E_{15} A SFRC/STS/ E_{15} B	P_{15}	3.44 3.48	5.12 5.20	6.92	1022	3.46	5.16
10	E_{23}	SFRC/ STS/E_{23} A SFRC/STS/ E_{23} B	P_{23}	3.02 3.13	4.75 4.82	6.15	9.57	3.08	4.79
11	E_{24}	SFRC/STS/ E_{24} A SFRC/STS/ E_{24} B	P_{24}	3.57 3.49	5.48 5.34	7.08	10.82	3.53	5.41
12	E_{25}	SFRC/STS/ E_{25} A SFRC/STS/ E_{25} B	P_{25}	3.23 3.28	3.82 3.86	6.51	7.68	3.26	3.84
13	E_{34}	SFRC/STS/ E_{34} A SFRC/STS/ E_{34} B	P_{34}	2.95 2.92	3.95 3.99	5.87	7.99	2.94	3.87

14	E_{35}	SFRC/ STS/E_{35} A SFRC/STS/ E_{35} B	P_{35}	3.12 3.14	4.25 4.30	6.26	8.55	3.13	4.28
15	E_{45}	SFRC/STS/ E_{45} A SFRC/STS/ E_{45} B	P_{45}	3.00 3.09	4.38 4.40	6.09	8.78	3.05	4.39

3.3. SFRC RESPONSES (FLEXURAL STRENGHT) FOR THE ECTP.

The response (Flexural strength) from the ECTP is shown in Table 5.

Table 5: SFRC Response (Flexural strength) For The ECTP

S/N	ECTP	EXPERIMENTAL NO.	RESPONSE MPa		Z_1	Z_2	Z_3	Z_4	Z_5	AVERAGE RESPONSE, MPa			
			14th day Results	28th day Results						14th day Results	28th day Results		
1	C_1	SFRC/FS/C_1 A SFRC/FS/ C_1 B	4.02 4.03	4.32 4.24	0.61	1	1.38	1.83	0.5	4.03	4.28	10.42	
2	C_2	SFRC/FS/ C_2 A SFRC/FS/ C_2 B	3.94 3.82	4.08 4.08	0.62	1	1.45	1.68	0.8	3.88	4.08	9.04	
3	C_3	SFRC/FS/ C_3A SFRC/FS/ C_3 B	5.24 5.28	6.10 6.15	0.67	1	1.4	1.7	1	5.26	6.13	7.33	
4	C_4	SFRC/ FS/C_4 A SFRC/FS/ C_4 B	5.11 5.17	6.28 6.30	0.66	1	1.3	1.68	1.2	5.14	6.29	7.89	
5	C_5	SFRC/FS/ C_5 A SFRC/FS/ C_5 B	5.28 5.17	5.24 5.28	0.63	1	1.28	1.63	1.5	5.23	5.26	12.81	
6	C_{12}	SFRC/FS/C_{12} A SFRC/FS/C_{12} B	5.23 5.15	5.48 5.49	0.64	1	1.36	1.7	0.65	5.19	5.49	10.77	
7	C_{13}	SFRC/FS/C_{13} A SFRC/FS/C_{13} B	4.89 4.82	6.00 5.98	0.59	1	1.45	1.83	0.75	4.86	5.99	7.6	
8	C_{14}	SFRC/FS/C_{14} A SFRC/FS/C_{14} B	4.26 4.38	4.30 4.32	0.59	1	1.48	1.77	0.85	4.30	4.31	8.1	
9	C_{15}	SFRC/FS/C_{15} A SFRC/FS/C_{15} B	4.31 4.33	4.86 4.88	0.65	1	1.42	1.8	1	4.30	4.87	7.05	
10	C_{23}	SFRC/FS/C_{23} A SFRC/FS/C_{23} B	3.99 4.09	4.92 4.97	0.64	1	1.3	1.77	0.9	4.04	4.95	7.25	
11	C_{24}	SFRC/FS/C_{24} A SFRC/FS/C_{24} B	6.21 6.23	7.00 7.02	0.6	1	1.27	1.71	1	6.22	7.01	8.04	

12	C_{25}	SFRC/FS/C_{25} A SFRC/FS/C_{25} B	5.30 5.31	6.92 6.94	0.6	1	1.31	1.79	1.15	5.31	6.93	7.96	
13	C_{34}	SFRC/FS/C_{34} A SFRC/FS/C_{34} B	4.19 4.23	6.80 6.82	0.62	1	1.33	1.83	1.1	4.21	6.81	8.14	
14	C_{35}	SFRC/FS/C_{35} A SFRC/FS/C_{35} B	4.38 4.42	6.22 6.21	0.63	1	1.41	1.85	1.25	4.40	6.22	10.54	
15	C_{45}	SFRC/FS/C_{45} A SFRC/FS/C_{45} B	6.18 6.23	5.82 5.91	0.61	1	1.25	1.79	1.35	6.21	5.87	11.02	

3.4. SFRC RESPONSES (SPLIT TENSILE STRENGHT) FOR THE ECTP.

The response (Split Tensile Strength) from the ECTP is shown in Table 6.

Table 6: SFRC Response (Split Tensile Strength) From The ECTP

S/N	**ECTP**	**EXPERIMENTAL NO.**	**RESPONSE MPa**		Z_1	Z_2	Z_3	Z_4	Z_5	**AVERAGE RESPONSE MPa**			
			14th day Results	**28th day Results**						**14th day Results**	**28th day Results**		
1	C_1	SFRC/STS/C_1 A SFRC/STS/C_1 B	2.24 2.26	3.82 3.72	0.61	1	1.38	1.83	0.5	2.25	3.77	10.42	
2	C_2	SFRC/STS/C_2 A SFRC/STS/C_2 B	2.11 2.05	3.52 3.48	0.62	1	1.45	1.68	0.8	2.08	3.50	9.04	
3	C_3	SFRC/STS/C_3 A SFRC/STS/C_3 B	2.95 2.98	3.72 3.75	0.67	1	1.4	1.7	1	2.97	3.74	7.33	
4	C_4	SFRC/STS/C_4 A SFRC/STS/C_4 B	3.56 3.62	3.88 3..82	0.66	1	1.3	1.68	1.2	3.59	3.85	7.89	
5	C_5	SFRC/STS/C_5 A SFRC/STS/C_5 B	2.85 2.87	4.20 4.30	0.63	1	1.28	1.63	1.5	2.86	4.25	12.81	
6	C_{12}	SFRC/STS/C_{12} A SFRC/STS/C_{12} B	2.48 2.52	4.86 4.84	0.64	1	1.36	1.7	0.65	2.50	4.85	10.77	
7	C_{13}	SFRC/STS/C_{13} A	3.22 3.19	4.92 5.02		1				3.21	4.97	7.6	

INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)
(Int Peer Reviewed Journal)
Vol. 04, Issue 09, September 2024, pp : 1116-1131
e-ISSN : 2583-1062
Impact Factor : 7.001

		SFRC/STS/ C_{13} B			0.59		1.45	1.83	0.75				
8	C_{14}	SFRC/STS/C_{14} A SFRC/STS/ C_{14} B	3.28 3.29	4.28 4.32	0.59	1	1.48	1.77	0.85	3.29	4.30	8.1	
9	C_{15}	SFRC/STS/ C_{15} A SFRC/STS/ C_{15} B	3.42 3.43	5.14 5.19	0.65	1	1.42	1.8	1	3.43	5.17	7.05	
10	C_{23}	SFRC/ STS/C_{23} A SFRC/STS/ C_{23} B	3..04 3.15	4.77 4.83	0.64	1	1.3	1.77	0.9	3.24	4.80	7.25	
11	C_{24}	SFRC/STS/ C_{24} A SFRC/STS/C_{24} B	3.58 3.52	5.47 5.31	0.6	1	1.27	1.71	1	3.55	5.39	8.04	
12	C_{25}	SFRC/STS/ C_{25} A SFRC/STS/ C_{25} B	3.25 3.29	3.84 3.80	0.6	1	1.31	1.79	1.15	3.27	3.82	7.96	
13	C_{34}	SFRC/STS/ C_{34} A SFRC/STS/ C_{34} B	2.94 2.90	3.95 3.92	0.62	1	1.33	1.83	1.1	2.92	3.94	8.14	
14	C_{35}	SFRC/ STS/C_{35} A SFRC/STS/ C_{35} B	3.14 3.19	4.34 4.32	0.63	1	1.41	1.85	1.25	3.17	4.33	10.54	
15	C_{45}	SFRC/STS/ C_{45} A SFRC/STS/ C_{45} B	3.09 3.12	4.30 4.42	0.61	1	1.25	1.79	1.35	3.11	4.36	11.02	

3.5. SCHEFFE' S (5,2) POLYNOMIAL MODEL FOR THE SFRC RESPONSES (FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT).

A. FLEXURAL STRENGHT

By substituting the values of the flexural strengths (responses) from Table 3 into Eqns.(8) through (10), we obtain the coefficients (β_1 , β_2 ... β_{34} ,β_{35}.... β_{45}) of the Scheffe's second degree polynomial for SFRC. Substituting the values of these coefficients into Eqn. (7) yields the polynomial model for the optimization of the flexural strength of SFRC (at both 14th day or 28th day) based on Scheffe's (5,2) lattice as stated under:

$$P^F = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 \quad \textbf{(22)}$$

B. SPLIT TENSILE STRENGHT

By substituting the values of the split tensile strengths (responses) from Table 4 into Eqns.(8) through (10), we obtain the coefficients (β_1 , β_2 ... β_{34} ,β_{35}.... β_{45}) of the Scheffe's second degree polynomial for SFRC. Substituting the values of these coefficients into Eqn. (7) yield the polynomial model for the optimization of the split tensile strength of SFRC (at both 14th day or 28th day) based on Scheffe's (5,2) lattice as given under:

$$P^S = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 \quad \textbf{(23)}$$

3.6. SCHEFFE'S (5,2) MODEL RESPONSES (FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT) FOR SFRC ECTP.

A. FLEXURAL STRENGHT

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4, C_5, … C_{45} of Table 5 into Eqn.(22), we obtain the Scheffe's second degree model responses (flexural strength) for the ECTP of SFRC.

B. SPLIT TENSILE STRENGHT

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4, C_5, … C_{45} of Table 6 into Eqn.(23), we obtain the second degree model responses (split tensile strength) for the ECTP of SFRC.

3.7. VALIDATION OF SFRC MODEL RESULTS (FOR FLEXURAL STRENGHT AND SPLIT TENSILE STRENGHT) USING STUDENT'S – T -TEST

In this session, the test of adequacy is performed to determine how correlated the SFRC flexural and split tensile strengths results (lab responses) given in Tables 5 and 6 and model responses from the control points based on Eqns.(22 and 23) are. Using the Student's – T – test as the means of validation, the result shows that there are no significant differences between the experimental results and model responses. The procedures involved in using the Student's – T - test have been explained by Nwachukwu and others (2022 c). Thus, the models are adequate for predicting the flexural and split tensile strengths of SFRC based on Scheffe's (5,2) simplex lattice.

3.8. RESULTS DISCUSSION

The maximum flexural strengths of SFRC based on Scheffe's (5,2) lattice are **7.06** MPa and **6.24** MPa respectively for 28^{th} and 14^{th} day results. Similarly the maximum split tensile strengths of SFRC based on Scheffe's (5,2) lattice are **5.42** MPa and **3.53** MPa respectively for 28^{th} and 14^{th} day results .The corresponding optimum mix ratio is **0.63:1:1.30:1.80:1.00** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Steel Fibre respectively. The minimum flexural strength and split tensile strength are **4.00** MPa, **3.90** MPa, **3.52**MPa and **2.10**MPa respectively for the 28^{th} day and 14^{th} day results. The minimum values correspond to the mix ratio of **0.56: 1:1.60:1.80: 0.80** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Steel Fibre respectively. Thus, the Scheffe's model can be used to determine the SFRC flexural and spilt tensile strength of all points (1 - 45) in the simplex based on Scheffe's Second Degree Model for five component mixture.

4. CONCLUSION

So far in this work, the Scheffe's Second Degree Polynomial (5,2) has been presented, discussed and used to formulate a model for predicting the flexural and split tensile strengths of SFRC. In the first instance, the Scheffe's model was used to predict the mix ratio for evaluating both the flexural and split tensile strengths of SFRC. Through the use of Scheffe's (5,2) simplex model, the values of both strengths were determined at all 15 points (1- 45). The result of the student's t-test shows that the strengths predicted by the models and the corresponding experimentally observed results are highly correlated. The maximum design strengths predicted by the model based on Scheffe's (5,2) model are as stated in the results discussion session, likewise the minimum values. Thus, with the Scheffe's (5,2) model, any desired strength of SFRC , given any mix ratio can be easily predicted and evaluated and vice versa. Thus, the application of this Scheffe's optimization model has reduced the problem of having to go through vigorous, time-consuming and laborious empirical mixture design procedures in order to obtain the desired design strengths of SFRC mixture.

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