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GAUSSIAN ANTI-MAGIC LABELING FOR A GRAPH AND DIGRAPH

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ABSTRACT

Graph labeling is one of the important research areas in graph theory. In this paper, we prove the existence of Gaussian anti-magic labeling for 4- regular $(n, 2n)$ graph of girth $j \geq 3$ and Cayley digraph associated with 2-generator 2- group graph. Also we investigate the existence of and Fibonacci cordial labeling for the Cayley digraph associated with 2- generator 2- group graph.

Keywords: Regular Graph, Cayley Digraph, 2-Generator 2-Group, Gaussian Anti-Magic Labeling, Fibonacci Cordial Labeling.

1. INTRODUCTION

DEFINITION

The concept of graph labeling was introduced by Rosa [5] in 1967. Graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. After the introduction of graph labeling, various labeling of graphs such as graceful labeling, magic labeling, anti-magic labeling, cordial labeling etc., have been studied in over 2500 papers [3]. In 1878, Cayley constructed a graph for a given group with a generating set which is now popularly known as Cayley graphs. A directed graph or digraph $G(V, E)$ consists of a finite set of points called vertices and a set of directed arrows between the vertices.

Let G be a finite group and S be a generating subset of G. The Cayley digraph denoted by $Cay(G, S)$, is the digraph whose vertices are the elements of G, and there is an arc from g to gs whenever $g \in G$ and $s \in S$. If $S = S^{-1}$ then there is an arc from g to gs if and only if there is an arc from gs to g. The Cayley graphs and Cayley digraphs are excellent models for interconnection networks [4]. For example, hypercube, butterfly, and cube-connected cycle's networks are Cayley graphs [1].

In 2018, a new type of graph labeling called Gaussian Anti-magic labeling has been introduced by K. Thirusangu and A. Selvaganapathy [8], Gaussian anti-magic labeling in a $G(p,q)$ graph is a function $f: V \to \{a + ib / a, b \in \mathbb{N}, 1 \leq a, b \leq q\}$ such that the induced function $f': E \to \mathbb{N}$ defined by $f'(e = uv) = |f(u)|^2 + |f(v)|^2$ where $|f(u)|^2 = a^2 + b^2$ if $f(u) = a + ib$, for all $u, v \in V$; $e \in E$, which results all the edge labels are distinct. A graph which admits Gaussian anti-magic labeling is called Gaussian anti-magic graph. Rokad and Ghodasara introduced a new labeling called Finonacci cordial labeling in 2016 [6], An Injective function $\eta: T(G) \longrightarrow \{\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_j\}$, where $\mathcal{F}_j$ is the j^{th} Fibonacci number $(j = 0,1, \ldots, l)$, is said to be Fibonacci cordial labeling if the induced function $\eta^*: W(G) \to \{0,1\}$ defined by $\eta^*(t_i t_j) = \big(\eta(t_i) + \eta(t_j)\big) \pmod 2$ satisfies the condition $\left|w_{\eta^*}(0) - w_{\eta^*}(1)\right| \leq 1$. A graph which admits Fibonacci cordial labeling is called Fibonacci cordial graph.

A group G is said to be a p-group if $o(G) = p^m, m \geq 1$. It is said to be 2-generated if the minimal generating set of G has exactly two elements. It is said to be a 2-group if $p = 2$. Cayley digraph for the 2-generated 2-group $Cay(G, (\alpha, \beta))$ has n vertices and $2n$ arcs with the vertex set of $Cay(G, (\alpha, \beta))$ as $V = \{v_1, v_2, v_3, \ldots, v_{n-1}, v_n\}$ and the arc set of $Cay(G, (\alpha, \beta))$ as $E(E_\alpha, E_\beta)$ where $E_\alpha = \{(v, \alpha v) | v \in V\}$ and $E_\beta = \{(v, \beta v) \mid v \in V\}$ or $E(E_\alpha', E_\beta')$ where $E_\alpha' = \{(\alpha v, v) \mid v \in V\}$ and $E_\beta' = \{(\beta v, v) \mid v \in V\}$. Denote the arcs in E_α as $\{g_\alpha(v_i) \mid v_i \in V\}$ and E_β as $\{g_\beta(v_i) \mid v_i \in V\}$ or E_α' as $\{g_\alpha'(v_i) \mid v_i \in V\}$ and E_β' as $\{g_\beta'(v_i) \mid v_i \in V\}$. Clearly each vertex in $Cay(G, (\alpha, \beta))$ has exactly two outgoing arcs out of which one arc is from the set E_α (E_α') and another is from the set E_β (E_β').

A graph G is said to be regular graph if degree of each vertex is equal. A graph is called k-regular if degree of each vertex in the graph G is k. It is said to be 4-regular graph if degree of each vertex in the graph G is 4. Girth of a graph G is defined as the length of smallest cycle. It is denoted by j. The possible girth j

$$= \begin{cases} 3 \leq j \leq \frac{n}{2} & \text{if } n \equiv 0 \pmod 2 \\ 3 \leq j \leq \frac{n+1}{2} & \text{otherwise} \end{cases}$$

4-regular graph with girth j has the vertex set and edge set as $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and

$$E = \left\{ \begin{array}{c} \{(v_k v_{k+1}),\ 1 \leq k \leq n-1\} \cup \{(v_k v_{k+(j-1)}),\ 1 \leq k \leq n-(j-1)\} \cup \\ \{(v_n v_1)\} \cup \{(v_{n-l} v_{j-(l+1)}), 1 \leq l \leq j-2\} \cup \{(v_n v_{j-1})\} \end{array} \right\}.$$ It is denoted by $(4\text{-RG})_{n,j}$.

2. MAIN RESULTS

In this section, we prove the existence of the gaussian anti-magic labeling for 4-regular graph with girth j and Cayley digraph associated with 2- generated 2- group. Also, we prove the existence of the Fibonacci cordial labeling for Cayley digraph associated with 2- generated 2- group.

THEOREM 2.1:

The graph $(4\text{-RG})_{n,j}$ admits Gaussian anti-magic labeling.

Proof:

From the construction, we have a 4-regular graph with girth j has n vertices and 2n edges.

Define the vertex set $f: V \to \{a + ib \,/\; a, b \in \mathbb{N}, 1 \le a, b \le 2n\}$ by

$$f(v_r) = r + i(2r + 1) \quad ; 1 \le r \le n$$

Vertex labelings are show in the table given below:

Vertex V	v_1	v_2	…	v_{n-1}	v_n
	$1 + i3$	$2 + i5$	…	$(n-1) + i\,(2n-1)$	$n + i(2n + 1)$

Define an induced function $f': E \to \mathbb{N}$ by

$f'(e = v_r v_s) = |f(v_r)|^2 + |f(v_s)|^2$ for all $v_r, v_s \in V, e \in E$.

Edge labelings obtained are shown in the table given below:

Edge E	v_1v_2	v_2v_3	…	$v_{n-1}v_n$
	39	87	…	$10n^2 - 2n + 3$
	v_1v_j	v_2v_{j+1}	…	$v_{n-j+1}v_n$
	$5j^2 + 4j + 11$	$5j^2 + 14j + 39$	…	$5(2n^2 + j^2) + 2n(9 - 5j) - 14j + 11$
	v_nv_1	$v_{n-l}v_{j-(l+1)}$		v_nv_{j-1}
	$5n^2 + 4n + 11$	$5[(n-l)^2 + (j-l)^2] + 2[2(n-l) - 3(j-l)] + 3$		$5(n^2 + j^2) + 2(2n - 3j) + 3$

Thus, all the edge labels are distinct.

Therefore, the graph $(4\text{-}RG)_{n,j}$ admits Gaussian anti-magic labeling.

EXAMPLE 2.1:

Gaussian anti-magic labeling for graph $(4\text{-}RG)_{9,3}$, graph $(4\text{-}RG)_{9,4}$, graph $(4\text{-}RG)_{9,5}$ is given in Figure 2.1, Figure 2.2, Figure 2.3, respectively.

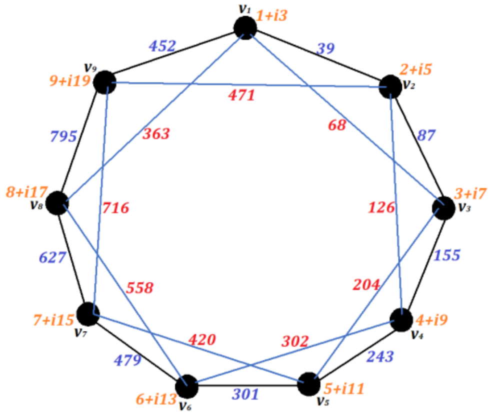

Figure 2.1: Gaussian anti-magic labeling for graph $(4\text{-}RG)_{9,3}$

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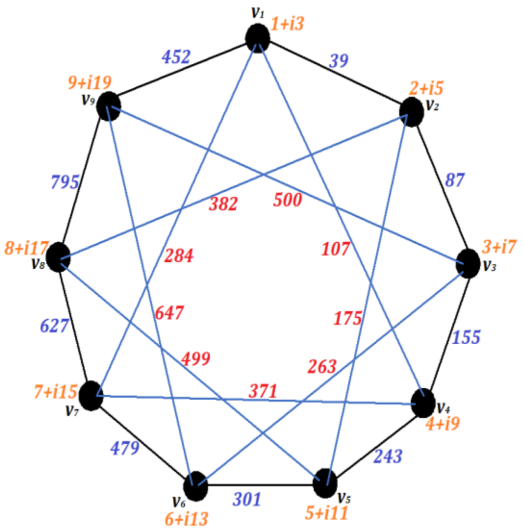

Figure 2.2: Gaussian anti-magic labeling for graph $(4\text{-}RG)_{9,4}$.

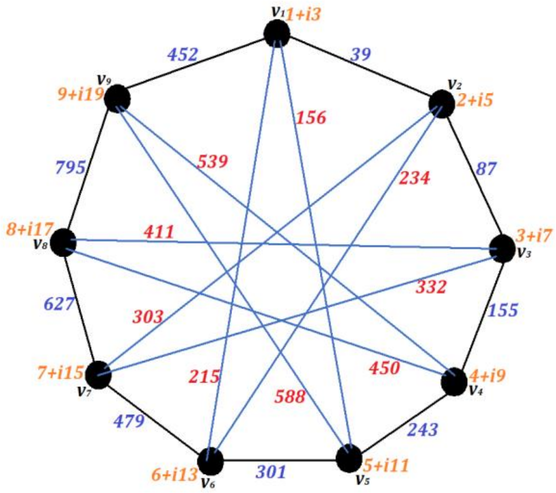

Figure 2.3: Gaussian anti-magic labeling for graph $(4\text{-}RG)_{9,5}$.

THEOREM 2.2:

The Cayley digraph associated with 2- generated 2- group admits Gaussian anti-magic labeling.

Proof:

From the structure of 2-generated 2-group $G = \{1, -1, p, -p, q, -q, r, -r\}$ with the generating set $S = \{p, q\}$ such that $p^2 = q^2 = r^2 = -1, pq = r, qr = p, rp = q, rp = -q, rq = -p, pr = -q$.

Let vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ be represented as $\{1, -1, p, -p, q, -q, r, -r\}$ respectively. Clearly, Cayley digraph associated with 2-generated 2-group G has 8 vertices and 16 edges.

Define a vertex set $f: V(G) \to \{a + ib \ / \ a, b \in \mathbb{N}, 1 \le a, b \le 16\}$ by

$$f(v_k) = k + i\,(k+1), \qquad 1 \le k \le 8.$$

Vertex labelings are show in the table given below:

Vertex V	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
	$1+i2$	$2+i3$	$3+i4$	$4+i5$	$5+i6$	$6+i7$	$7+i8$	$8+i9$

Define an induced function $f': E(G) \to \mathbb{N}$ by

$f'(e = v_k v_j) = |f(v_k)|^2 + |f(v_j)|^2$ for all $v_k, v_j \in V, e \in E$.

Edge labelings obtained are shown in the table given below:

Edge E				
	v_1v_3	v_1v_4	v_1v_5	v_1v_6
	30	46	66	90
	v_2v_3	v_2v_4	v_2v_5	v_2v_6
	38	54	74	98
	v_3v_7	v_3v_8	v_4v_7	v_4v_8
	138	170	154	186
	v_5v_7	v_5v_8	v_6v_7	v_6v_8
	174	206	198	230

Thus, all the edge labels are distinct.

Therefore, Cayley digraph associated with 2- generated 2- group admits Gaussian anti-magic labeling.

EXAMPLE 2.2:

Gaussian anti-magic labeling for Cayley digraph associated with 2-generated 2-group is given in Figure 2.4.

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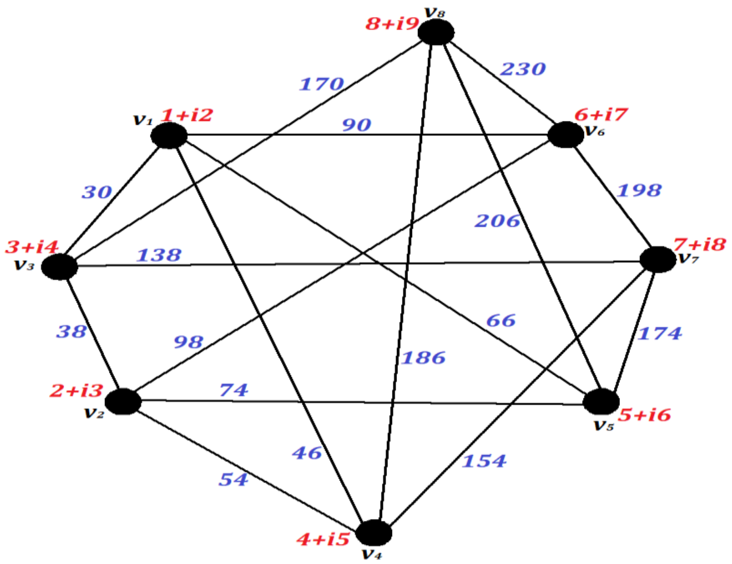

Figure 2.4: Gaussian anti-magic labeling for Cayley digraph associated with 2-generated 2-group.

THEOREM 2.3:

The Cayley digraph associated with 2- generated 2- group admits Fibonacci cordial labeling.

Proof:

We know that, Cayley digraph associated with 2-generated 2-group G has 8 vertices and 16 edges.

Define a vertex set $f: V(G) \to \{\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_4, \mathcal{F}_5, \mathcal{F}_6, \mathcal{F}_7, \mathcal{F}_8\}$ by

$$f(t_i) = \mathcal{F}_{i-1}, \qquad 1 \leq i \leq 2$$

$f(t_i) = \mathcal{F}_i,\ 3 \leq i \leq 8.$

Define an induced function $f^*: W(G) \to \{0,1\}$ by

$$f'(t_i t_j) = \left(f(t_i) + f(t_j)\right)\ (mod\ 2)$$

Thus, $E_{f'}(0) = 8; E_{f'}(1) = 8,\ |E_{f'}(0) - E_{f'}(1)| = |8 - 8| = 0.$

Clearly, the condition $|E_{f'}(0) - E_{f'}(1)| \leq 1$ is satisfied.

Therefore, Cayley digraph associated with 2- generated 2- group admits Fibonacci cordial labeling.

EXAMPLE 2.3:

Fibonacci cordial labeling for Cayley digraph associated with 2-generated 2-group is given in Figure 2.5.

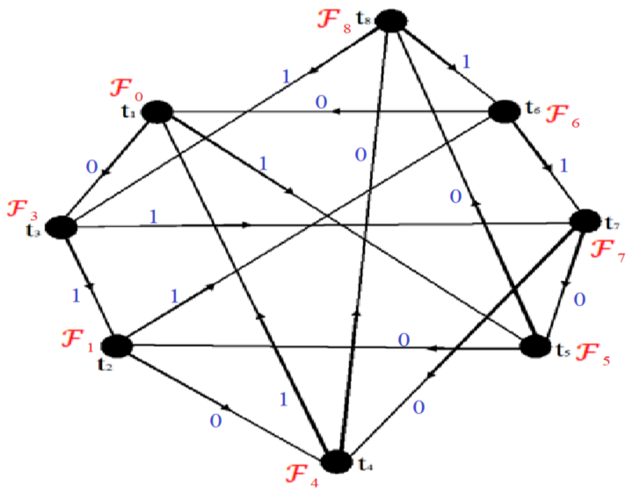

Figure 2.5: Fibonacci cordial labeling for Cayley digraph associated with 2-generated 2-group.

3. CONCLUSION

In this paper we have proved the existence of Gaussian anti-magic labeling for 4- regular $(n, 2n)$ graph of girth $j \geq 3$ where $n \geq 5$ and Cayley digraph associated with 2- generator 2- group graph. Also, we investigated the existence of Fibonacci cordial labeling for the Cayley digraph associated with 2- generator 2- group graph.

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