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ON THE LOMAX-DAGUM-X FAMILY OF DISTRIBUTION

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ABSTRACT

In this research work, we construct a new class of asymmetric continuous probability distributions (Lomax-Dagum-X family), which serve as extensions of the Dagum-X family distribution. However, the density function and the cumulative distribution function of the constructed distribution (Lomax-Dagum-X family distribution) were obtained. We also, test the validity of the modified asymmetric probability distribution with corresponding plots of the distribution.

Keywords: Lomax-G, Dagum-X family

1. INTRODUCTION

Statistical distributions are commonly applied to describe real-world phenomena. Due to the usefulness of statistical distributions, their theory is widely studied and new distributions are developed. The interest in developing more flexible statistical distributions remains strong in the statistics profession. Many generalized classes of distributions have been developed and applied to describe various phenomena. A common feature of these generalized distributions is that they have more parameters. Johnson et al. (1994) stated that the use of four-parameter distributions should be sufficient for most practical purposes. According to these authors, at least three parameters are needed but they doubted any noticeable improvement arising from including a fifth or sixth parameter.

Statistical distributions find extensive application in describing real-world occurrences. Given their utility, the theory behind statistical distributions is extensively researched, leading to the development of novel distributions. The quality of statistical distribution is based on fitting the assumed probability distribution to the data. However, there are various issues where any of these distributions do not fit the data appropriately, especially in engineering, finance, medicine, and environmental hazards. Therefore, a significant effort has been made in developing different families of distributions Amani et al., (2023). The statistical profession continues to show a keen interest in creating even more adaptable distribution models. Statistical models play a crucial role in representing and analyzing datasets in practical applications. While traditional distributions, such as Weibull, Lomax, Uniform, gamma, log-normal, exponential, and beta, have been widely used, they may not always provide a satisfactory fit for complex datasets. Researchers have been actively developing new asymmetrical models offering greater adaptability and flexibility to address this limitation. These advancements often involve techniques, such as exponentiation, transformation, modification, T–X, beta, and gamma generator approaches, to generate more flexible asymmetrical distributions Sapkota et al., (2023).

The choice of appropriate distributions to be used on real-life data plays a fundamental role in improving the power, efficiency, and sensitivity of statistical tests. This is so because appropriate distributions lead to a good fit of the data. Therefore, good knowledge of the appropriate distribution to be used for a specific data set is essential. Probability distributions are very important in data analysis. They can be used to model a wide range of data shapes in applied fields. The Lomax (Lx) (also known as Pareto II) distribution has many applications in several areas such as income and wealth inequality, biological sciences, lifetime and reliability, engineering, and actuarial sciences. The Lx distribution has been applied in modeling real-life data in income and wealth, firm size, and reliability and life testing Atkinson and Harrison (1978); Corbellini et al., (2010); Harris (1968); Hassan and Al-Ghamdi (2009). Chahkandi and Ganjali (2009) showed that the Lx distribution belongs to the decreasing hazard rate (HR) family. More information about the Lx distribution can be explored in Anorld (1983); Johnson et al., (1994); Tahir et al., (2015). The procedure of adding new shape parameters for generalizing classical distributions is a well-known technique in the statistical literature. Hence, there are several extensions of the Lx distribution which are developed using well-known families to improve its flexibility and applicability in modeling different types of data. For example, Gupta et al. (1998) introduced the exponentiated Lomax, the Marshall–Olkin Lomax was proposed by Ghitany et al., (2007), the Kumaraswamy–Lomax by Lemonte and Cordeiro (2013), the Weibull–Lomax was studied by Tahir et al., (2015), the exponentiated half logistic-Lomax was introduced by Afify et al., (2017), the Fréchet Topp–Leone Lomax was proposed by Reyad et al., (2021), and the generalized linear failure rate Lomax by Afify et al., (2022). Recently, numerous classes of continuous probability/lifetime models have been proposed by different studies in the statistical literature. It has been proved that they are useful for adding performance, skewness, and flexibility of properties to the existing models. In other words, all of these approaches extend the classical baseline probability distributions by introducing additional parameter(s) to the baselines, thereby making the extended baselines much more flexible to fit a wide range of data from the practical situation. Some of these classical distributions include the Dagum-X family distribution by Amani et al., (2023) among others. However, real complex datasets characterized by asymmetric (kurtosis, skewness, N-shape, W-shapes etc) and bathtub in nature cannot be fitted/model with the aforementioned distributions as the estimate obtain from such distributions will be characterized by higher dispersion/variation. To overcome this challenge, some new distributions that are adaptive and flexible to accommodate datasets that are asymmetrical or bathtub in nature are required taking non-asymmetric distribution as baseline distribution. Therefore, in this research, some new distributions with the potential to accommodate asymmetrical or bathtub datasets will be proposed. Specifically, a new family distribution called Lomax-Dagum-G family of distribution will be proposed. Dagum (1977) proposed the distribution which is referred to as Dagum distribution which is based on the log logistic distribution by adding another parameter.

It is also called the generalized logistic-Burr distribution. There is both a three-parameter specification (Type I) and a four-parameter specification (Type II) of the Dagum distribution. Lukasiewicz et al. (2010) made a comparison among four models with various numbers of parameters: exponential, Weibull, Dagum, and Singh-Maddala to determine which model can represent the data that comes from the personal incomes in the USA by using some of the important measures such as the sum of squared residuals, the sum of absolute values of the residuals. Tahir et al., (2016) proposed a Weibull Dagum distribution where Its density function is very flexible and can be symmetrical, left-skewed, right-skewed, and reversed-J shaped. It has constant, increasing, decreasing, upside-down bathtub, bathtub, and reversed-J-shaped hazard rate.

Various structural properties are derived including explicit expressions for the quantile function, ordinary and incomplete moments, and probability-weighted moments. Tahir et al., (2016) proposed a Weibull Dagum distribution where Its density function is very flexible and can be symmetrical, left-skewed, right-skewed, and reversed-J shaped. It has constant, increasing, decreasing, upside-down bathtub, bathtub, and reversed-J-shaped hazard rate. Various structural properties are derived including explicit expressions for the quantile function, ordinary and incomplete moments, and probability-weighted moments. Rodrigues and Silva (2015) proposed Gamma Dagum distribution. application of the gamma-Dagum distribution to real data shows that the new distribution can be used quite effectively to provide better fits than the beta-Dagum, beta-Pareto, and Pareto confluent hypergeometric distributions. Oluyede and Rajasooriya (2013) proposed a new class of distributions called the Mc-Dagum distribution. An important motivation for the development Mc-Dagum distribution is the benefit of this class in its ability to fit skewed data that cannot properly be fitted in many other existing distributions.

2. METHOD FOR GENERATING A NEW FAMILY OF DISTRIBUTION

Let $r(t)$ be the pdf of a random variable $T \in [a,b]$, for $-\infty \le a \le b \le \infty$. Let $W(F(x))$ be a function of the cdf $F(x)$ of any random variable X so that $W(F(x))$ satisfies the following conditions.

1. $W(F(x)) \in [a,b]$
2. $W(F(x))$ is differentiable and monotonically non-decreasing
3. $W(F(x)) \to a$ as $x \to -\infty$ and $W(F(x)) \to b$ as $x \to \infty$.

Let X be the random variable with PDF $f(x)$ and CDF $F(x)$.

$$F(x) = \int_a^{W(F(x))} r(t)dt \qquad (1)$$

$$f(x) = \left[\frac{d}{dx} W(F(x))\right] r\left[W(F(x))\right] \qquad (2)$$

The definition of $W(F(x))$ depends on the support of the random variable T as follows:

1. When the support T is bounded: $W(F(x))$ can be defined as $F(x)$ or $F(x)^a$.

2. when the support T is $[a,\infty]$, for $a\geq 0$: $W(F(x))$ can be defined as $-\log(1-F(x))$ or $F(x)/(1-F(x))$ or $-\log(1-F(x)^{a})$.
3. When the support of T is $(-\infty,\infty)$: $W(F(x))$ can be define as $\log\left[-\log(1-F(x))\right]$ or $\log\left[F(x)/(1-F(x))\right]$ by Alzaatreh et al., (2013).

2.1 Dagum-X Family of distribution

Amani et al., (2023) proposed a generalized Dagum distribution using the T-X method by Alzaatreh et al., (2013). The new family of Dagum distribution called Dagum-X, can be defined as follows: The CDF and PDF of the proposed New Dargum-X family of distribution by Amani et al. (2023) as in (3) and (4) respectively.

$$G(x;\theta,\lambda,\delta)=\left(1+\lambda\left(\frac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta} \quad (3)$$

The PDF is obtained by differentiating equation (3) concerning X as follows $g(x;\theta,\delta,\lambda)=\beta\theta\delta g(x)\frac{[G(X)]^{-\delta-1}}{[\bar{G}(X)]^{-\delta+1}}\left(1+\lambda\left(\frac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta-1}$ (4)

Where $x>0$ and $\delta,\theta>0$ are the shape parameters and $\lambda>0$ is the scale parameter respectively.

The survival function $S(x;\theta,\delta,\beta)$ and hazard rate function $h(x;\theta,\delta,\beta)$ of the Dagum-X family are obtained as in (5) and (6)respectively.

$$S(x;\theta,\lambda,\delta)=1-\left[1-\left(1+\lambda\left(\frac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta}\right] \quad (5)$$

$$h(x;\theta,\lambda,\delta)=\frac{\lambda\theta\delta g(x)\frac{[G(X)]^{-\delta-1}}{[\bar{G}(X)]^{-\delta+1}}\left(1+\lambda\left(\frac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta-1}}{1-\left[1-\left(1+\lambda\left(\frac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]} \quad (6)$$

Where $\lambda>0$, is the scale parameter and $\delta,\theta>0$ are the shape parameters respectively.

2.2 New Lomax -G Family

Sakpota et al., (2023) proposed a family of distribution called the New Lomax-G family of distribution which serves as the extension of Pareto type II (Lomax) distribution so that its support begins at zero by Lomax (1954).

The CDF and PDF of the proposed new family New Lomax-G Family distribution is given in (7) and (8) respectively.

$$G(x;\alpha,\beta)=\beta^{\alpha}\left[\beta-\log\left(F(x)\right)\right]^{-\alpha} \quad \text{and,} \quad (7)$$

$$g(x;\alpha,\beta)=\alpha\beta^{\alpha}f(x)F(x)^{-1}\left[\beta-\log\left(F(x)\right)\right]^{-(\alpha+1)} \quad (8)$$

Where $\alpha>0$ and $\beta>0$ are shape and scale parameters respectively.

Tthe survival function $S(x;\alpha,\beta)$ and hazard rate function $h(x;\alpha,\beta)$ of the lomax-x family is given in equation (9) and (10) respectively.

$$S(x;\alpha,\beta)=1-\beta^{\alpha}\left[\beta-\log\left(F(x)\right)\right]^{-\alpha} \quad (9)$$

$$h(x;\alpha,\beta)=\frac{\alpha\beta^{\alpha}f(x)F(x)^{-1}\left[\beta-\log\left(F(x)\right)\right]^{-(\alpha+1)}}{1-\beta^{\alpha}\left[\beta-\log\left(F(x)\right)\right]^{-\alpha}} \quad (10)$$

Where $x>0$ $\alpha>0$ and $\beta>0$ are the Shape and Scale parameters respectively.

2.3 The Proposed Lomax-Dagum-G Family Distribution

Then, the New Lomax-Dagum-G family has the cdf given by (11).

$$F(x;\alpha,\beta,\delta,\theta,\lambda)=\beta^{\alpha}\left[\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]^{-\alpha} \qquad x>0, \quad (11)$$

Where $\overline{G}(x)=1-G(x)$, $\delta>0$ and $\theta>0$ are the shape parameters, and $\lambda>0$ is the scale parameter.

To obtain the corresponding PDF, we differentiate (11) with respect to x and obtain (12)

$$\frac{d}{dx}F(x;\delta,\lambda,\theta)=f(x;\delta,\lambda,\theta)$$

$$y=\left[\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]^{-\alpha} \text{ by keeping the } \beta^{\alpha} \text{ as a constant}$$

$$f(x;\alpha,\beta,\theta,\delta,\lambda)=\alpha\theta\delta\lambda g(x)\frac{G(x)^{-\delta-1}}{\overline{G}(x)^{-(\delta+1)}}\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-1}\left[\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]^{-\alpha-1} \quad (12)$$

Where $\delta>0$, $\alpha>0$, $\beta>0$, and $\theta>0$ are the shape parameters, and $\lambda>0$ is the scale parameter.

4 Model Validity Check of the Proposed Lomax-Dagum-G family of distribution

To ensure the proposed probability distribution function (PDF) of the new family is valid and correct, it must satisfy the fact that;

$$\int_0^{\infty} f(x;\alpha,\beta,\theta,\delta,\lambda)=1 \quad (13)$$

The proof

$$\int_0^{\infty}\alpha\beta^{\alpha}\theta\delta\lambda g(x)\frac{G(x)^{-\delta-1}}{\overline{G}(x)^{-(\delta+1)}}\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-1}\left[\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]^{-\alpha-1} dx=1 \quad (14)$$

Let

$$y=\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}, \quad k=\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}, \quad w=1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}, \quad t=\frac{G(x)}{\overline{G}(x)}$$

$$y=\beta-\log k, \; k=w^{-\theta}, \quad w=1+\lambda t^{-\delta} \text{ and } \frac{dt}{dx}=\frac{g(x)}{\overline{G}(x)^{2}}$$

By substituting the values of k, w and t we have

$$dx=-\frac{1}{\theta\delta\lambda g(x)G(x)^{-\delta-1}\overline{G}(x)^{-(\delta+1)}\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-1}}dy$$

We then substitute for dx in (3.24) and given (3.25) respectively.

$$=-\int_0^{\infty}\alpha\beta^{\alpha}\left[\beta-\log k\right]^{-\alpha-1}dk \quad (15)$$

Recall that $k=\left(1+\lambda\left(\frac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta}$

Then we simplify the (3.25) at when $x\to\infty$, and when $x\to 0$

$$\int_0^\infty f(x;\alpha,\beta,\theta,\delta,\lambda)=-\alpha\beta^{\alpha}\frac{\left[\beta^{-\alpha}-\log k^{-\alpha}\right]_0^\infty}{-\alpha} \tag{16}$$

$$\int_0^\infty f(x;\alpha,\beta,\theta,\delta,\lambda)=(1-0)-(1-1)=1 \tag{17}$$

Hence, the model in equation (12) is a valid probability density function.

2.4 Plots CDF and PDF of Lomax-Dagum X family

Plots of cumulative distribution function and probability density function of Lomax-Dagum X family for selected/varying values for the five parameter to study its behaviour are given in Figures 1 and 2 and furthermore, for a similarity, $a=\alpha, b=\beta, c=\delta, d=\theta$ *and* $e=\lambda$.

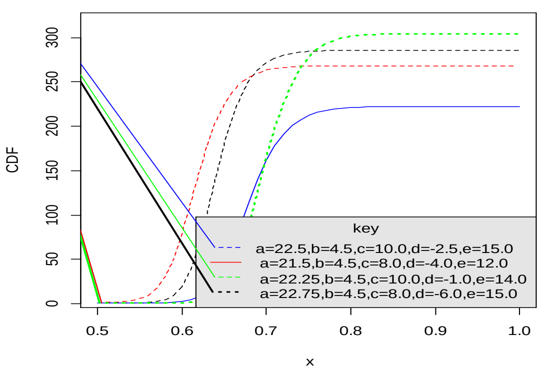

Figure 1.

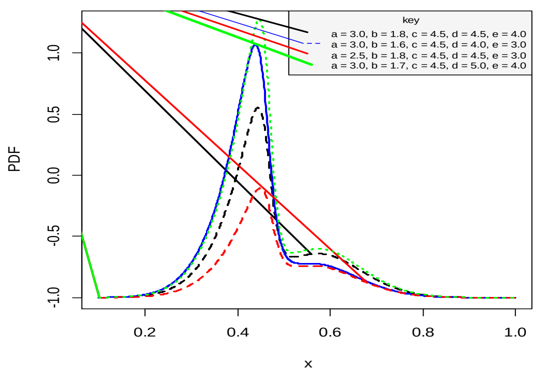

Figure 2.

Figure (1) displays the distribution function of LD-X family for different values of the shape and scale parameters. Figure (2) displays the density function of LD-X family for different values of the shape and scale parameters. It is both left and right skewed distribution and has different level kurtosis which shows the flexibility of the distribution for modeling asymmetric datasets

3. CONCLUSION

In this paper, we construct a new classes of asymmetric continuous probability distributions (Lomax-Dagum-X family) which serve as extension of Dagum-X family. However, density function and the cumulative distribution function of the constructed distribution (Lomax-Dagum-X family were obtained. We also, proceed to test the validity of the new asymmetric distribution. Explicit expressions for some basic statistical properties of these distributions such as Moments, Moment Generating Function, Quantile function and order statistics was derived.

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