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MODELING AND ANALYSIS OF SERIES CIRCUITS USING LINEAR DIFFERENTIAL EQUATIONS

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ABSTRACT

Differential equations originate from the mathematical formulation of a great variety of problems in science and engineering. In this paper, we discussed the application of Linear Differential Equations to series circuits. First we formulated the problem mathematically by applying Kirchhoff's voltage law, thereby obtaining a differential equation. Then we solved the equation and interpreted the solution which finds the current at time $t > 0$,in terms of the quantities involved in the problem and also graphically.

Keywords: Linear Differential equation, Kirchhoff's Voltage Law, Resistor, Inductor, Electromotive Force, Time and Current.

1. INTRODUCTION

The application of Linear Differential Equation to series circuits containing (1) an electromotive force, and (2) resistors, inductors, and capacitors.

Electromotive Force: A battery or generator produces a flow of current in a closed circuit and that this current produces a so-called voltage drop across each resistor, inductor, and capacitor.

This is measured in volt(v). The letter "E" is used in equations.

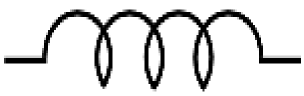
The symbol for electromotive force as shown below.

RESISTORS: A resistor represents a given amount of resistance in a circuit. Resistance is a measure of how the flow of electric current is opposed or "resisted." It is defined by Ohm's law which says the resistance equals the voltage divided by the current. Resistance = voltage/current   R = v/i

Resistance is measured in Ohms. The Ohm is often represented by the omega symbol: Ω. The symbol for resistance is a zigzag line as shown below. The letter "R" is used in equations.

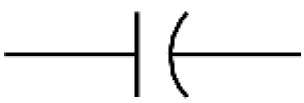
Resistor Symbol

Inductors: An inductor represents the amount of inductance in a circuit. The inductance is the ability of a component to generate electromotive force due to a change in the flow of current. A simple inductor is made by looping a wire into a coil. Inductors are used in electronic circuits to reduce or oppose the change in electric current. Inductance is measured in Henrys. The symbol for inductance is a series of coils as shown below. The letter "L" is used in equations.



Inductor Symbol

Capacitors: A capacitor represents the amount of capacitance in a circuit. The capacitance is the ability of a component to store an electrical charge. The capacitance is defined by the equation c = q/v where q is the charge in coulombs and v is the voltage. Capacitance is measured in Farads. The symbol for capacitance is two parallel lines. Sometimes one of the lines is curved as shown below. The letter "C" is used in equations.



Capacitor Symbol

Further, the following three laws concerning the voltage drops across these various elements are known to hold:

1. The voltage drop across a resistor is given by $E_R = Ri$ where R is a constant of proportionality called the resistance, and i is the current.
2. The voltage drop across an inductor is given by $E_L = L\frac{di}{dt}$ where L is a constant of proportionality called the inductance, and i again denotes the current.
3. The voltage drop across a capacitor is given by $E_c = \frac{1}{c}q$ where c is a constant of proportionality called the capacitance and q is the instantaneous charge on the capacitor.

Since $i = \frac{dq}{dt}$,this is often written as $E_c = \frac{1}{c}\int i\, dt$.

The fundamental law in the study of electric circuits is the following:

KIRCHHOFF'S VOLTAGE LAW (FORM 1): The algebraic sum of the instantaneous voltage drops around a close circuit in a specific direction is zero. Since voltage drops across resistors, inductors and capacitors have the opposite sign from voltages arising from electromotive forces, we may state this law in the following alternative form.

KIRCHHOFF'S VOLTAGE LAW(FORM 2): The sum of the voltage drops across resistors, inductors, and capacitors is equal to the total electromotive force in a closed circuit.

We now consider the circuit shown in figure 1.1

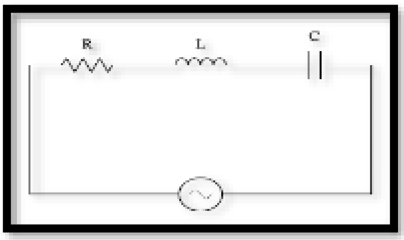


Figure 1.1

Let us apply Kirchhoff's law to the circuit of figure 1.1. Letting E denote the electromotive force, and using the laws 1,2 and 3 for voltage drops that were given above, we are led at once to the equation

$L\frac{di}{dt} + Ri + \frac{1}{c}q = E$ …………(I)

This equation contains two dependent variables i and q.

If the circuit contains no capacitor, equation (I) itself reduces directly to

$L\frac{di}{dt} + Ri = E$ ……………(II)

2. LINEAR DIFFERENTIAL EQUATION OF FIRST ORDER

A differential equation of the form $\frac{dy}{dx} + Py = Q$ ……….(III) where P and Q are functions of x only is called a linear differential equation of the first order in y.

The general solution of the linear differential equation (III) is

$$y\ (\text{Integrating Factor}) = \int Q\ (\text{Integrating Factor})\, dx + k$$

Where Integrating Factor $= e^{\int P dx}$

STATEMENT OF THE PROBLEM: A circuit has in series an electromotive force given by $E = 150\cos 200t\ V$, a resistor of 10Ω and an inductor of 0.2H.If the initial current is 0 then finding the current at time $t > 0$.

FORMULATION:

Let i denote the current in amperes at time t.

The total electromotive force is $150\cos 200t\ V$.

Using the laws 1 and 2, we find that the voltage drops are as follows:

1. Across the resistor: $E_R = Ri = 10i$
2. Across the inductor: $E_L = L\frac{di}{dt} = \frac{1}{5}\frac{di}{dt}$

Applying Kirchhoff's law, have the differential equation

$$\frac{1}{5}\frac{di}{dt} + 10i = 150\cos 200t$$

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$\frac{di}{dt} + 50i = 750\cos 200t$ ………………(1)

Since the initial current is 0, the initial condition is i(0)=0……(2)

3. APPLYING LINEAR DIFFERENTIAL EQUATION TO THE PROBLEM:

Equation (1) is a first order linear differential equation.

An integrating factor is $e^{\int 50\,dt} = e^{50t}$

General Solution of (1) is

$$i\,(\text{Integrating Factor}) = \int 750\cos 200t\ (\text{Integrating Factor})dt + k$$

$$i\,e^{50t} = \int 750\cos 200t\ \ e^{50t}\,dt + k$$

$$i\,e^{50t} = 750\int \cos 200t\ \ e^{50t}\,dt + k$$

$$ie^{50t} = 750\,\frac{e^{50t}}{2500+40000}\,[50\cos 200t + 200\sin 200t] + k$$

$$i\,e^{50t} = 750\,\frac{e^{50t}}{42500}\,50\,[\cos 200t + 4\sin 200t] + k$$

$$i\ = 750\,\frac{e^{50t}}{850}\ [\cos 200t + 4\sin 200t]e^{-50t} + k\,e^{-50t}$$

$i\ = \frac{15}{17}\ [\cos 200t + 4\sin 200t] + k\,e^{-50t}$ …………(3)

Applying the conditions (2), i=0 when t=0 we get $k = -\frac{15}{17}$

Thus the solution is $i\ = \frac{15}{17}\ [\cos 200t + 4\sin 200t] - \frac{15}{17}e^{-50t}$

Expressing the trigonometric terms in a "phase-angle" form, we have

$$i = \frac{15}{17}\sqrt{17}\left(\frac{1}{\sqrt{17}}\cos 200t + \frac{4}{\sqrt{17}}\sin 200t\right) - \frac{15}{17}e^{-50t}$$

$i = \frac{15}{\sqrt{17}}\sin(200t + \phi) - \frac{15}{17}e^{-50t}$ ……..(4)

Where ϕ is determined by the equation

$$\phi = \arccos\frac{4}{\sqrt{17}} = \arcsin\frac{1}{\sqrt{17}}$$

We find $\phi \approx 0.24\text{rad}$, and hence the current is given approximately by

$i = 3.64\sin(200t + 0.24) - 0.88\,e^{-50t}$ ………(5)

TABLE:1

Time(t)	Current(i)
0	-0.01476
0.1	3.5750
0.2	2.0573
0.3	-1.9017
0.4	-3.6095
0.5	-1.0442
0.6	2.7573
0.7	3.2946
0.8	-0.0683
0.9	-3.3504
1.0	-2.6661

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4. RESULT

The current in equation (5) is clearly expressed as the terms of a sinusoidal term and an exponential. The exponential becomes so very small in a short time that its effect is soon practically negligible; it is the transient term. Thus, after a short time, essentially all that remains is the sinusoidal term; it is the steady-state current. Observed that its period $\frac{\pi}{100}$ is the same as that of the electromotive force. However, the phase angle $\phi \approx 0.24$ indicates that the electromotive force leads the steady-state current.

GRAPHICAL INTERPRETATION:

The graph of the current as a function of time appears in the figure 1.2

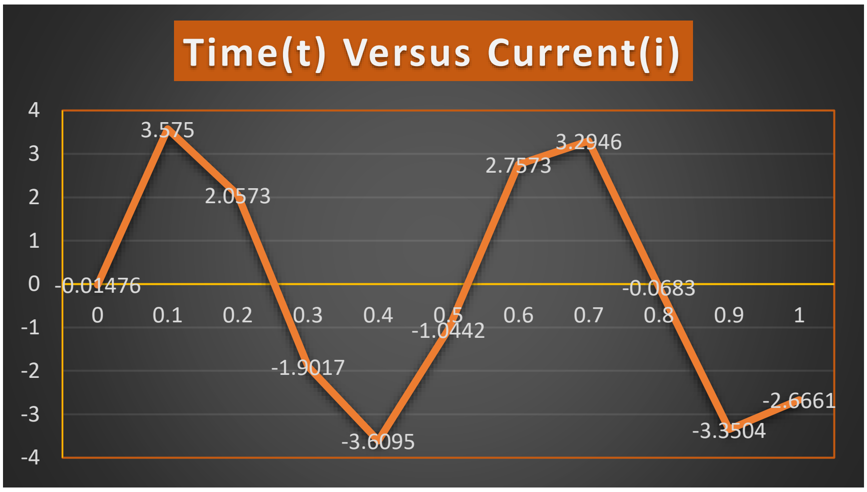

Figure 1.2

5. CONCLUSION

In this paper, we have applied first order linear differential equation to series circuits containing an electromotive force, resistor, inductor and capacitor by applying Kirchhoff's voltage law. Finally, the equation $i = 3.64\sin(200t + 0.24) - 0.88\,e^{-50t}$ which was useful to calculate the current i approximately at any time t.

In this paper, we have successfully applied the Kirchhoff's law modified into second order linear differential equation to series circuits containing an electromotive force, resistor, inductor and capacitor.

We assume that the reader is somewhat familiar with these items. We have been simply recalled that the electromotive force produces a flow of current in a closed circuit and the current produces a so-called voltage drop across each resistor, inductor, and capacitor.

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