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# ON FINDING INTEGRAL SOLUTIONS OF TERNARY QUADRATIC EQUATION

## $x^2+y^2=z^2-10$

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## ABSTRACT

This paper illustrates the process of obtaining different sets of non-zero distinct integer solutions to the non-homogeneous ternary quadratic Diophantine equations given by $x^2+y^2=z^2-10$.

**Keywords:** **N**on-Homogeneous Quadratic , Ternary Quadratic, Integer Solutions.

## 1. INTRODUCTION

It is known that Diophantine equations with multidegree and multiple variables are rich in variety[1,2]. While searching for the collection of second degree equations with three unknowns, the authors came across the papers [3-12]in which the authors obtained integer solutions to the ternary quadratic equations $x^2+y^2=z^2+N, N=1,\pm4,\pm8,12,-2k^2,10,-12$ The above papers motivated us for obtaining non zero distinct integer solutions to the above equation for other values to N. This communication illustrates process of obtaining different sets of non-zero distinct integer solutions to the non-homogeneous ternary quadratic Diophantine equation given by $x^2+y^2=z^2-10$ .

## 2. METHOD OF ANALYSIS

The non-homogeneous ternary quadratic Diophantine equation under consideration is

$$x^2+y^2=z^2-10 \qquad (1)$$

The process of obtaining different sets of integer solutions to (1) is illustrated below:

**Illustration 1:**

The choice

$$z=x+h, h\geq 0 \qquad (2)$$

in (1) leads to the parabola

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$$y^2=2hx+h^2-10 \qquad (3)$$

It is possible to choose $h, x$ so that the R.H.S. of (3) is a perfect square and the value of $y$ is obtained. Substituting the values of $h, x$ in (2),the corresponding value of $z$ satisfying (1) is obtained. For simplicity and brevity, a few examples are given in Table 1 below:

Table 1 : Examples

| h | x | y | z |
|---|---|---|---|
| 1 | $2k^2-2k+5$ | $2k-1$ | $2k^2-2k+6$ |
| 5 | $10k^2-10k+1$ | $10k-5$ | $10k^2-10k+6$ |

**Illustration 2:**

The substitution of the linear transformations

$$z=(2k^2-2k+6)s, x=(2k^2-2k+5)s \qquad (4)$$

in (1) leads to the negative pell equation

$$y^2=(4k^2-4k+11)s^2-10 \qquad (5)$$

for which the integer solutions exist when k takes particular values.

Example :1

Considering the value of k to be 1 in (4),it gives the negative pell equation

$$y^2 = 11s^2 - 10 \tag{6}$$

After some algebra ,the corresponding integer solutions to (6) are given by

$$y_{n+1} = (\frac{1}{2}f_n + \frac{\sqrt{11}}{2}g_n) \tag{7}$$

$$s_{n+1} = (\frac{1}{2}f_n + \frac{1}{2\sqrt{11}}g_n) \tag{8}$$

where $f_n = (10+3\sqrt{11})^{n+1} + (10-3\sqrt{11})^{n+1}$ , $g_n = (10+3\sqrt{11})^{n+1} - (10-3\sqrt{11})^{n+1}$

Using (8) in (4), one obtains that

$$x_{n+1} = 5(\frac{1}{2}f_n + \frac{1}{2\sqrt{11}}g_n), z_{n+1} = 6(\frac{1}{2}f_n + \frac{1}{2\sqrt{11}}g_n) \tag{9}$$

Thus,(7) and (9) represent the integer solutions to (1).

Example :2

Considering the value of k to be 2 in (4),it gives the negative pell equation

$$y^2 = 19s^2 - 10 \tag{10}$$

After some algebra ,the corresponding integer solutions to (10) are given by

$$y_{n+1} = (\frac{3}{2}f_n + \frac{\sqrt{19}}{2}g_n) \tag{11}$$

$$s_{n+1} = (\frac{1}{2}f_n + \frac{3g_n}{2\sqrt{19}}) \tag{12}$$

where $f_n = (170+39\sqrt{19})^{n+1} + (170-39\sqrt{19})^{n+1}$ , $g_n = (170+39\sqrt{19})^{n+1} - (170-39\sqrt{19})^{n+1}$

Using (12) in (4), one obtains that

$$x_{n+1} = 9(\frac{1}{2}f_n + \frac{3g_n}{2\sqrt{19}}) , z_{n+1} = 10(\frac{1}{2}f_n + \frac{3g_n}{2\sqrt{19}}) \tag{13}$$

Thus,(11) and (13) represent the integer solutions to (1).

**Illustration 3:**

The substitution of the linear transformations

$$z = u + h , x = u - k , u \neq k \neq 0, h \neq k \neq 0 \tag{14}$$

in (1) leads to

$$y^2 = 2(h+k)u + h^2 - k^2 - 10 \tag{15}$$

Remember that $h, k$ are non-zero distinct integers and it is possible to choose them

Such that the R.H.S. of (15) is a perfect square and the value of $y$ is obtained. Substituting the values of $u, h, k$ in (14), the corresponding values of $x, z$ are found . A few numerical examples are exhibited in Table 1: below:

Table 1: Examples

| h | k | x | y | z |
|---|---|---|---|---|
| 3 | 2 | $10s^2 - 10k + 1$ | $10s - 5$ | $10s^2 - 10k + 6$ |
| 5 | 2 | $14s^2 - 18s + 3$<br>$14s^2 - 10s - 1$ | $14s - 9$<br>$14s - 5$ | $14s^2 - 18s + 10$<br>$14s^2 - 10s + 6$ |

**Illustration 4:**

The substitution of the linear transformations

$$y = kx, k \neq 1 \tag{16}$$

in (1) leads to the positive pell equation

$$z^2 = (k^2 + 1)\,x^2 + 10 \tag{17}$$

for which the integer solutions exist when k takes particular values.

Example :3

Considering the value of k to be 3 in (16),it gives the positive pell equation

$$z^2 = 10x^2 + 10 \tag{18}$$

After some algebra ,the corresponding integer solutions to (18) are given by

$$x_{n+1} = \frac{3}{2} f_n + \frac{\sqrt{10}}{2} g_n \tag{19}$$

$$z_{n+1} = \frac{1}{2}(10 f_n + 3\sqrt{10}\, g_n) \tag{20}$$

where

$$f_n = (19+6\sqrt{10})^{n+1} + (19-6\sqrt{10})^{n+1} \;, g_n = (19+6\sqrt{10})^{n+1} - (19-6\sqrt{10})^{n+1}$$

In view of (16), we have

$$y_{n+1} = \frac{3}{2}(3 f_n + \sqrt{10}\, g_n) \tag{21}$$

Thus,(19) ,(20) and (21) represent the integer solutions to (1).

Example :4

Considering the value of k to be 5 in (16),it gives the positive pell equation

$$z^2 = 26x^2 + 10$$

After some algebra ,the corresponding integer solutions to (1) are given by

$$x_{n+1} = \frac{1}{2} f_n + \frac{3}{\sqrt{26}} g_n$$

$$z_{n+1} = \frac{1}{2}(6 f_n + \sqrt{26}\, g_n)$$

$$y_{n+1} = 5(\frac{1}{2} f_n + \frac{3}{\sqrt{26}} g_n)$$

where

$$f_n = (51+10\sqrt{26})^{n+1} + (51-10\sqrt{26})^{n+1} \;, g_n = (51+10\sqrt{26})^{n+1} - (51-10\sqrt{26})^{n+1}$$

**Illustration 5:**

The substitution of the linear transformation

$$x = y + h, h \neq 0 \tag{22}$$

in (1) leads to the negative pell equation

$$(2y + h)^2 = 2\,z^2 - (h^2 + 20) \tag{23}$$

for which the integer solutions exist when h takes particular values.

Example :5

Considering the value of h to be 4 in (23),it gives the negative pell equation

$$(2y + 4)^2 = 2\,z^2 - 36 \tag{24}$$

In this case, the corresponding integer solutions to (24) are given by

$$y_{n+1} = \frac{1}{2}(3f_n + \frac{6}{\sqrt{2}} g_n - 4) \quad (25)$$

$$z_{n+1} = \frac{1}{2}(6f_n + 3\sqrt{2} g_n) \quad (26)$$

where

$$f_n = (3+2\sqrt{2})^{n+1} + (3-2\sqrt{2})^{n+1}, g_n = (3+2\sqrt{2})^{n+1} - (3-2\sqrt{2})^{n+1}$$

Using (25) in (22), one obtains that

$$x_{n+1} = \frac{1}{2}(3f_n + \frac{6}{\sqrt{2}} g_n + 4) \quad (27)$$

Thus,(25),(26) and (27) give the integer solutions to (1).

Example :6

Considering the value of h to be 6 in (23),it gives the negative pell equation

$$(2y+6)^2 = 2z^2 - 56 \quad (28)$$

In this case, the corresponding integer solutions to (28) are given by

$$y_{n+1} = \frac{1}{2}(2f_n + 3\sqrt{2} g_n - 6) \quad (29)$$

$$z_{n+1} = (3f_n + \sqrt{2} g_n) \quad (30)$$

where

$$f_n = (3+2\sqrt{2})^{n+1} + (3-2\sqrt{2})^{n+1}, g_n = (3+2\sqrt{2})^{n+1} - (3-2\sqrt{2})^{n+1}$$

Using (29) in (22), one obtains that

$$x_{n+1} = \frac{1}{2}(2f_n + 3\sqrt{2} g_n + 6) \quad (31)$$

Thus,(29),(30) and (31) give the integer solutions to (1).

## 3. CONCLUSION

In this paper ,an attempt has been made to obtain different sets of non-zero distinct integer solutions to the ternary quadratic diophantine equation $x^2 + y^2 = z^2 - 10$. As diophantine equations are rich in variety ,the readers of this paper may search for choices of the integer solutions to the other forms of ternary quadratic diophantine equations.

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