INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)
(Int Peer Reviewed Journal)
Vol. 05, Issue 02, February 2025, pp : 71-77
e-ISSN : 2583-1062
Impact Factor : 7.001
www.ijprems.com
editor@ijprems.com

SOME DIVISOR CORDIAL LABELINGS OF STAR RELATED GRAPH

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DOI: https://www.doi.org/10.58257/IJPREMS38441

ABSTRACT

In this paper, we investigate the existence of some divisor cordial labeling for star graph.

keywords: Graph labeling, Star graph, Triplicate graph, Divisor cordial labeling.

1. INTRODUCTION

In 1967, the concept of graph labeling was introduced by Rosa[8] . A graph labeling is an assignment of integers to the edges or vertices or both to the certain conditions. In 2011[1], the concept of the extended triplicate graph of a path P_p was introduced by Bala and Thirusangu. In 2023[2], the concept of Extended triplicate graph of star ETG($k_{1,p}$) was introduced by Bala .et.al.,. The concept of Square divisor cordial labeling was introduced by Murugesan[9]. Let G be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a square divisor cordial labeling if an induced function S^*: β(G) → {0,1} defined by S^*: β (G) → {0,1} by $S^*(bc) = \begin{cases} 1\,; if\ ((s(c))^2|s(b))\ or (s(c)|(s(b))^2) \\ 0\,; \quad otherwise \end{cases}$,∀ bc ∈ β(G) satisfies$|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$. A graph which admits a square divisor cordial labeling is called as square divisor cordial graph. The concept of Sum divisor cordial labeling was introduced by Lourdusamy.et.al.,[7]. Let G be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a sum divisor cordial labeling if an induced function S^*: β(G) → {0,1} defined by $S^*(bc) = \begin{cases} 1\,;\ if (2|(s(b) + s(c))) \\ 0\,; \quad otherwise \end{cases}$, ∀ bc ∈ β(G) satisfies$|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$. A graph which admits a Sum divisor cordial labeling is called as sum divisor cordial graph. The concept of Subtract divisor cordial labeling and multiply divisor cordial labeling was introduced by Gondalia.et.al.,[5,6]. Let G be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a subtract divisor cordial labeling if an induced function S^*: β(G) → {0,1} defined by $S^*(bc) = \begin{cases} 1\,;\ if (2|(s(b) - s(c))) \\ 0\ ; \quad otherwise \end{cases}$, ∀ bc ∈ β(G) satisfies$|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$. A graph which admits a Subtract divisor cordial labeling is called as subtract divisor cordial graph. Let G be a simple graph with p vertices and q edges. A bijective function S: δ(G) → $\{1, 2, 3, \dots\dots, p\}$ is said to be a multiply divisor cordial labeling. if an induced function S^*: β(G) → {0,1} defined by $S^*(bc) = \begin{cases} 1\ ;\ if (2|(s(b).s(c))) \\ 0\,; \quad otherwise \end{cases}$,∀ bc ∈ β(G) satisfies the condition $|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$. A graph which admits a Multiply divisor cordial labeling is called as Multiply divisor cordial graph.

By stimulation of the above studies, In this paper we investigate the existence of Square divisor cordial labeling, Sum divisor cordial labeling, Subtract divisor cordial labeling, and Multiply divisor cordial labeling in the context of Extended Triplicate graph of star.

2. MAIN RESULT

In this section, we investigate the existence of Square divisor cordial labeling, Sum divisor cordial labeling, Subtract divisor cordial labeling and Multiply divisor cordial labeling for the Extended Triplicate graph of star.

2.1 STRUCTURE OF EXTENDED TRIPLICATE OF STAR GRAPH

Let G be a star graph ($K_{1,p}$).The triplicate graph of star with vertex set δ'(G) and edge set β'(G) is given by: δ'(G) = $\{b \cup b' \cup b'' \cup c_i \cup c_i' \cup c_i''/1 \le i \le$ p } and β'(G) = $\{bc_i' \cup b'c_i \cup b'c_i'' \cup b''c_i'/1 \le i \le p\}$. Clearly, Triplicate graph of star TG($K_{1,p}$) with this vertex set and edge set is disconnected. To make this a connected graph, include a new edge b$c_1$to the edge set β'(G) . Thus, we get an **Extended Triplicate of star graph** with vertex $set\ \delta = \delta'$and edge set $\beta(G) = \beta'$(G) ∪ bc_1 . Clearly, ETG($K_{1,p}$) has $3(p + 1)$ vertices and $(4p + 1)$ edges and is denoted by ETG($K_{1,p}$).[2].

Theorem 2.1: Extended triplicate of star graph is a Sum divisor cordial graph.

Proof: Extended Triplicate of star graph ETG($K_{1,p}$) has vertex set δ(G) = $\{b \cup b' \cup b'' \cup c_i \cup c_i' \cup c_i''/1 \le i \le$ p } and edge set βG) = $\{bc_i' \cup b'c_i \cup b'c_i'' \cup b''c_i' \cup$ b$c_1/1 \le i \le p\}$

Clearly, it has $3(p + 1)$ vertices and $(4p + 1)$ edges.

To show that ETG($K_{1,p}$) is a Sum divisor cordial graph.

Define the bijective function S: δ(G) → $\{1, 2, 3, \ldots\ldots, 3(p+1)\}$ to label the vertices as follows.

$if, p = 3$	$S(b) = 2$	$S(b') = 1$	$S(b'') = 3$
	For, $1 \le i \le p$	$S(c_i) = 2i + p$	$S(c_i') = 2(i+1)$
	For, $2 \le i \le p$	$S(c_1'') = 2(p+1) + 3$	$S(c_i'') = 2(p+i)$
$if, p > 3$	$S(b) = 3$	$S(b') = 1$	$S(b'') = 2$
	For, $1 \le i \le p$	$S(c_i) = 2i + 3$	$S(c_i'') = 2(i+1)$
	To find $S(c_i')$		
	$p \equiv 0\ mod2$	$S(c_i') = \begin{cases} 2(p+i)+3 \ ; 1 \le i \le \frac{p}{2} \\ p + 2(i+1) \ ; \frac{p+2}{2} \le i \le p \end{cases}$	
	$p \equiv 1\ mod2$	$S(c_i') = \begin{cases} 2(p+i)+3 \ ; 1 \le i \le \frac{p-1}{2} \\ p + 2i + 3 \ ; \frac{p+1}{2} \le i \le p \end{cases}$	

Define an induced function S^*: β(G) → {0,1} by $S^*(bc) = \begin{cases} 1 \ ; \ if(2|(s(b)+s(c))) \\ 0 \ ; \ otherwise \end{cases}$,

∀ bc ∈ β(G) to obtain the edge labels as follows.

$if, p = 3$	For, $2 \le i \le p$	$S^*(b'c_i'') = 0$	$S^*(bc_1) = 0$
	For, $1 \le i \le p$	$S^*(b''c_i') = 0$	$S^*(b'c_i) = S^*(bc_i') = 1$
$if, p > 3$	$S^*(bc_1) = 1$		
	For, $1 \le i \le p$	$S^*(b'c_i) = 1$	$S^*(b'c_i'') = 0$
	To find $S^*(bc_i')$ and $S^*(b''c_i')$		
	$p \equiv 0\ mod2$	$S^*(bc_i') = \begin{cases} 1 ; \ 1 \le i \le \frac{p}{2} \\ 0 ; \frac{p+2}{2} \le i \le p \end{cases}$	$S^*(b''c_i') = \begin{cases} 0 ; \ 1 \le i \le \frac{p}{2} \\ 1 ; \frac{p+2}{2} \le i \le p \end{cases}$
	$p \equiv 1\ mod2$	$S^*(bc_i') = \begin{cases} 1 ; \ 1 \le i \le \frac{p-1}{2} \\ 0 ; \frac{p+1}{2} \le i \le p \end{cases}$	$S^*(b''c_i') = \begin{cases} 0 ; \ 1 \le i \le \frac{p-1}{2} \\ 1 ; \frac{p+1}{2} \le i \le p \end{cases}$

Thus, we get

For, $p = 3$	$\beta_{S^*}(0) = 2p$	$\beta_{S^*}(1) = 2p + 1$	$\lvert\beta_{S^*}(0) - \beta_{S^*}(1)\rvert = \lvert 2p - (2p+1)\rvert \le 1$
For, $p > 3$	$\beta_{S^*}(0) = 2p$	$\beta_{S^*}(1) = 2p + 1$	$\lvert\beta_{S^*}(0) - \beta_{S^*}(1)\rvert = \lvert 2p - (2p+1)\rvert \le 1$

From both the cases, it is clear that $|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$ is satisfied.

Hence, the Extended triplicate of star graph is a sum divisor cordial graph.

EXAMPLE 2.1: ETG($K_{1,3}$) ,ETG($K_{1,5}$) and its Sum divisor cordial labelling is shown in figure 1 and figure 2.

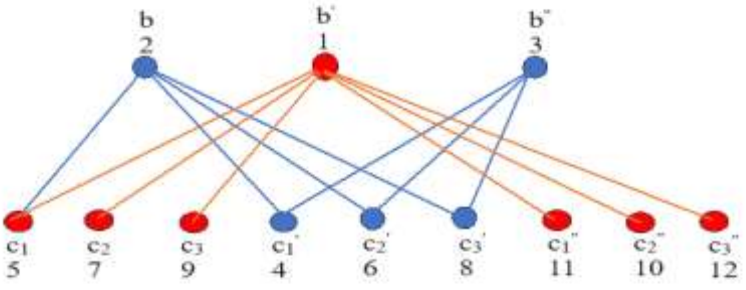

FIGURE – 1

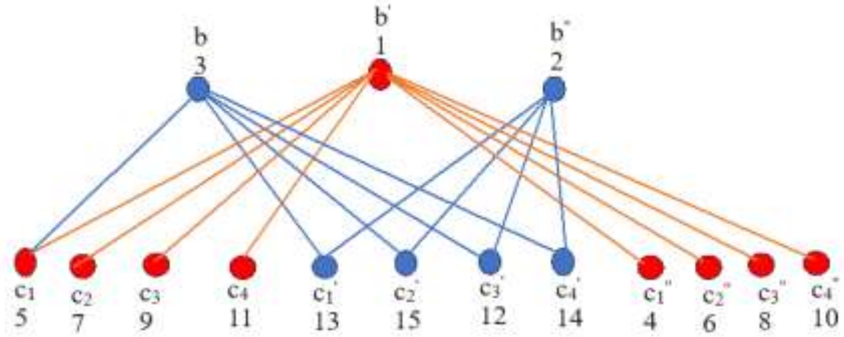

FIGURE – 2

Theorem 2.2: Extended triplicate of star graph is a Subtract divisor cordial graph.

Proof: Extended Triplicate of star graph ETG($K_{1,p}$) has $3(p+1)$ vertices and $(4p+1)$ edges.

To show that ETG($K_{1,p}$) is a Subtract divisor cordial graph.

Define the bijective function $S:\delta(G) \to \{1,2,3,\ldots\ldots,3(p+1)\}$ to label the vertices as follows.

$if, p = 3$	$S(b) = 3p$	$S(b') = 3p+2$	$S(b'') = 3(p+1)$
	For, $1 \le i \le p$	$S(c_i) = 2i$	$S(c_i'') = p + 2(i-1)$
	$S(c_i') = \begin{cases} 2(p+i) & ; \quad 1 \le i \le \frac{p+1}{2} \\ 1 & ; \quad i = \frac{p+3}{2} \end{cases}$		
$if,$ $p \equiv 1 \bmod 2$ $(p \neq 3)$	$S(b) = 3p$	$S(b') = 3p+2$	$S(b'') = 3(p+1)$
	For, $1 \le i \le p$	$S(c_i) = 2i$	$S(c_i'') = p + 2(i-1)$
	$S(c_i') = \begin{cases} 2(p+i); & 1 \le i \le \frac{p+1}{2} \\ 1 \quad ; & i = \frac{p+3}{2} \\ s(c'_{\frac{p+3}{2}}) + 2j; & \frac{p+2j+3}{2} \le i \le p ; j \epsilon N \end{cases}$		
$if,$ $p \equiv 0 \bmod 2$	$S(b) = 3p+1$	$S(b') = 3(p+1)$	$S(b'') = 3p+2$
	For, $1 \le i \le p$	$S(c_i) = 2i$	$S(c_i'') = p + 2i - 1$
	$S(c_i') = \begin{cases} 2(p+i); & 1 \le i \le \frac{p}{2} \\ 1 \quad ; & i = \frac{p+2}{2} \\ s(c'_{\frac{p+2}{2}}) + 2j \quad ; & \frac{p+2(j+1)}{2} \le i \le p ; j \epsilon N \end{cases}$		

Define an induced function as $S^*: \beta(G) \to \{0,1\}$ by $S^*(bc) = \begin{cases} 1; & if\ (2|(s(b)-s(c))) \\ 0\ ; & otherwise \end{cases}$, $\forall$ bc $\in \beta(G)$ to obtain the edge labels as follows.

$S^*(bc_1) = 0$		
For, $1 \le i \le p$	$S^*(b'c_i) = 0$	$S^*(b'c_i'') = 1$
To find $S^*(bc_i') and\ S^*(b''c_i')$		
$if,$ $p \equiv 0 \bmod 2$	$S^*(bc_i') = \begin{cases} 0; & 1 \le i \le \frac{p}{2} \\ 1; & \frac{p+2}{2} \le i \le p \end{cases}$	$S^*(b''c_i') = \begin{cases} 1; & 1 \le i \le \frac{p}{2} \\ 0; & \frac{p+2}{2} \le i \le p \end{cases}$
$if,$ $p \equiv 1 \bmod 2$	$S^* bc_i') = \begin{cases} 0\ ; 1 \le i \le \frac{p+1}{2} \\ 1\ ; \frac{p+3}{2} \le i \le p \end{cases}$	$S^*(b''c_i') = \begin{cases} 1 & ; 1 \le i \le \frac{p+1}{2} \\ 0 & ; \frac{p+3}{2} \le i \le p \end{cases}$

From the above labeling, we get $\beta_{S^*}(0) = 2p+1$ and $\beta_{S^*}(1) = 2p$

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e-ISSN : 2583-1062
Impact Factor : 7.001

Thus $|\beta_{S^*}(0) - \beta_{S^*}(1)| = |(2p+1) - 2p| \leq 1$. It is clear that $|\beta_{S^*}(0) - \beta_{S^*}(1)| \leq 1$ is satisfied.

Hence, Extended Triplicate of star graph is a Subtract divisor cordial graph.

EXAMPLE 2.2: ETG($K_{1,3}$), ETG($K_{1,4}$) and ETG($K_{1,5}$) and its Subtract divisor cordial labelling is shown in figure 3, figure 4 and figure 5.

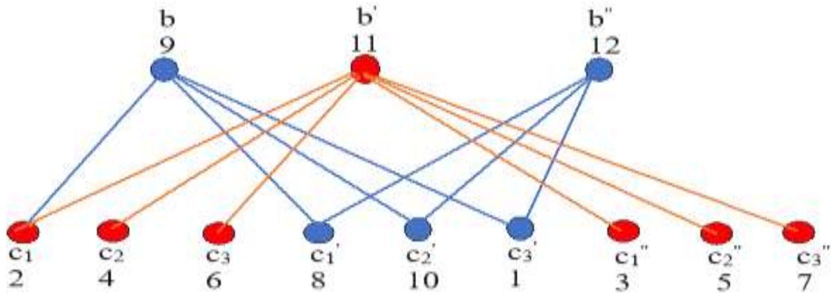

FIGURE – 3

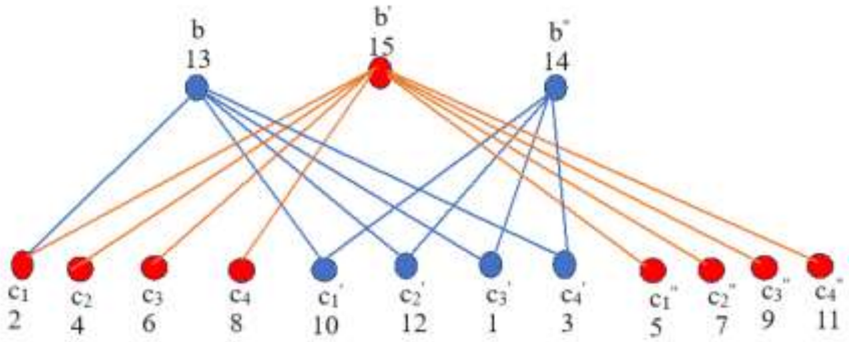

FIGURE – 4

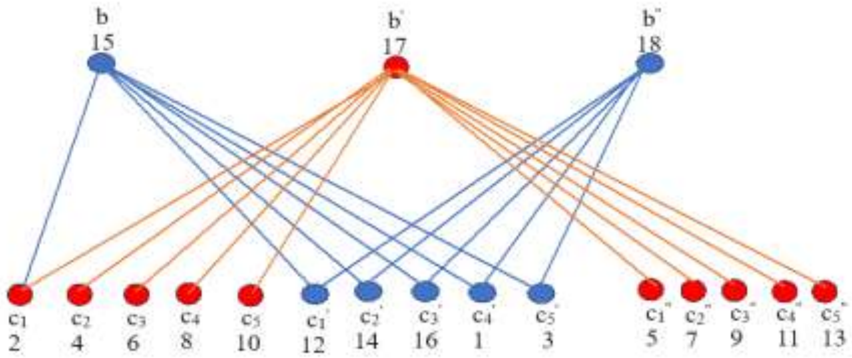

FIGURE – 5

Theorem 2.3: Extended triplicate of star graph is a Multiply divisor cordial graph.

Proof: Extended Triplicate of star graph ETG($K_{1,p}$) has $3(p+1)$ vertices and $(4p+1)$ edges.

To show that ETG($K_{1,p}$) is a Multiply divisor cordial graph.

Define the bijective function S: $\delta(G) \rightarrow \{1, 2, 3, \ldots\ldots, 3(p+1)\}$ to label the vertices as follows.

$S(b) = 1$	$S(b') = 2$	$S(b'') = 3$
For, $1 \leq i \leq p$	$S(c_i) = 2(i+1)$	$S(c_i') = 2i+3$
To find $S(c_i'')$		
$if, p \equiv 1 \, mod \, 2$	$S(c_i'') = \begin{cases} 2(p+i+1) & ; 1 \leq i \leq \frac{p+1}{2} \\ p+2(i+1) & ; \frac{p+3}{2} \leq i \leq p \end{cases}$	

$if, p \equiv 0 \; mod \; 2$	$S(c_i'') = \begin{cases} 2(p+i+1) \; ; & 1 \le i \le \frac{p}{2} \\ p+2i+3 \; ; & \frac{p+2}{2} \le i \le p \end{cases}$

Define an induced function $S^*: \beta\,(G) \to \{0,1\}$ by $S^*(bc) = \begin{cases} 1 \; ; & if\,(2|(s(b).s(c))) \\ 0 \; ; & otherwise \end{cases}$,∀ bc ∈ β(G) to obtain the edge labels as follows.

For, $1 \le i \le p$	$S^*(bc_1) = 1$	$S^*(b'c_i) = 1$	$S^*(\,b'c_i'') = 1$
	$S^*(\,bc_i'\,) = 0$	$S^*(b''c_i') = 0$	

We get $\beta_{S^*}(0) = 2p$ and $\beta_{S^*}(1) = 2p + 1$

Thus, $|\beta_{S^*}(0) - \beta_{S^*}(1)| = |2p - (2p+1)| \le 1$

Here, it is clear that $|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$ is satisfied.

Hence, the Extended Triplicate of star graph is a Multiply divisor cordial graph.

EXAMPLE 2.3: ETG($K_{1,4}$), ETG($K_{1,5}$) and its Multiply divisor cordial labelling is shown in figure 6 and figure 7.

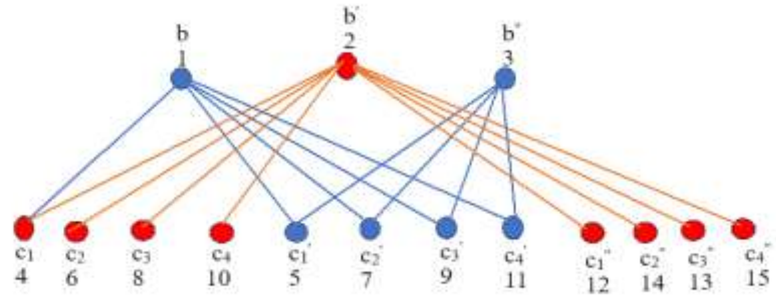

FIGURE – 7

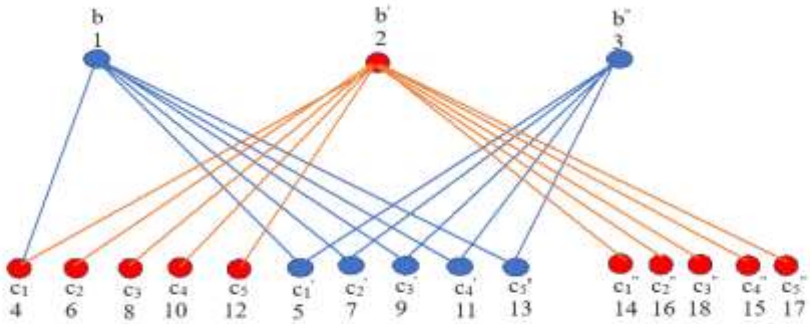

FIGURE – 8

Theorem 2.4: Extended triplicate of star graph is a square divisor cordial graph.

Proof: Extended Triplicate of star graph ETG($K_{1,p}$) has $3(p + 1)$ vertices and $(4p + 1)$ edges.

To show that ETG($K_{1,p}$) is a Square divisor cordial graph.

Define the bijective function $S: \delta(G) \to \{1, 2, 3, \ldots\ldots, 3(p+1)\}$ to label the vertices as follows.

$if,$ $p = 3$	$S(b) \;=\; 2$	$S(b') = 3$	$S(b'') = 1$
	For, $1 \le i \le p$	$S(c_i) = 2(i+1)$	$S(c_i') = 2i + 3$
	For, $1 \le i \le p - 1$	$S(c_i'') = 2(p + i + 1)$	$S(c_p''\,) = 4p - 1$
$if,$ $p > 3$	$S(b\,) \;=\; 2$	$S(b') = 1$	$Sb'') = 3$
	For,$2 \le i \le p$	$S(c_i) = 2(i+1)$	$S(c_1) = 5$
	$S(c_1') = 7$	$S(c_i') = \begin{cases} 2i+7 \; ; & 2 \le i \le 6 \\ 2(i+j)+7 \; ; & 2j+5 \le i \le 2j+6 \; ; j \in N \end{cases}$	
	To find $S(c_i'')$		

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<table>
<tr><td rowspan="4"></td><td rowspan="2">$for, p \equiv 0 \; mod \; 2$</td><td colspan="2">$S(c_i'') = \begin{cases} 2(p+i+1) \; ; & 1 \le i \le \frac{p}{2} \\ 21 \; ; & i = \frac{p+2}{2}, for \; p > 4 \\ c''_{\frac{p+2(j-1)}{2}} + 6 \; ; & \frac{p+2j}{2} \le i \le p-2 \; ; j \epsilon N - \{1\}, for \; p > 6 \end{cases}$</td></tr>
<tr><td>$S(c''_{p-1}) = 9$</td><td>$S(c''_p) = S(c''_{p-1}) - 5$</td></tr>
<tr><td rowspan="2">$for, p \equiv 1 \; mod \; 2$</td><td colspan="2">$S(c_i'') = \begin{cases} 2(p+i+1) \; ; & 1 \le i \le \frac{(p+1)}{2} \\ 21 \; ; & i = \frac{p+3}{2}, for \; p > 5 \\ c''_{\frac{p+2j-1}{2}+6} \; ; & \frac{p+1+2j}{2} \le i \le p-2 \; ; j \epsilon N - \{1\}, for \; p > 7 \end{cases}$</td></tr>
<tr><td>$S(c''_{p-1}) = 9$</td><td>$S(c''_p) = S(c''_{p-1}) - 5$</td></tr>
</table>

Define an induced function as $S^*: \beta \; (G) \to \{0,1\}$ by $S^*(bc) = \begin{cases} 1 \; ; if \; ((s(c))^2 | s(b)) \; or (s(c)|(s(b))^2) \\ 0 \; ; \quad otherwise \end{cases}$,∀ bc ∈ β(G) to get the edge labels.

We get

$if, p = 3$	$\beta_{S^*}(0) = 2p+1$	$\beta_{S^*}(1) = 2p$	$\lvert\beta_{S^*}(0) - \beta_{S^*}(1)\rvert = \lvert 2p+1-2p \rvert \le 1$
$if, p > 3$	$\beta_{S^*}(0) = 2p$	$\beta_{S^*}(1) = 2p+1$	$\lvert\beta_{S^*}(0) - \beta_{S^*}(1)\rvert = \lvert 2p-(2p+1) \rvert \le 1$

From both the cases, it is clear that $|\beta_{S^*}(0) - \beta_{S^*}(1)| \le 1$ is satisfied.

Hence, Extended triplicate of star graph is a square divisor cordial graph.

EXAMPLE 2.4: ETG($K_{1,3}$), ETG($K_{1,4}$) and its square divisor cordial labelling is shown in figure 9 and figure 10.

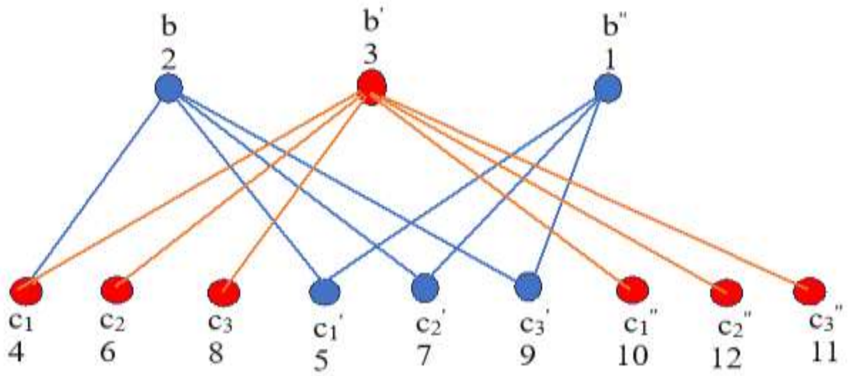

FIGURE – 9

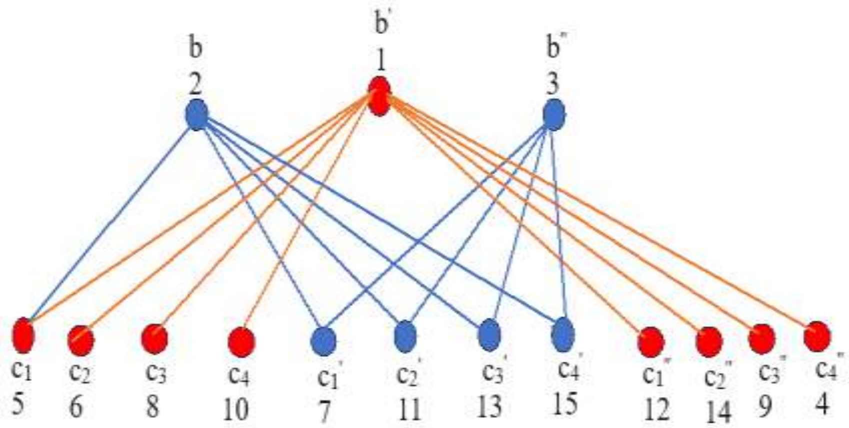

FIGURE – 10

3. CONCLUSION

In this paper, we have proved that extended triplicate of star graph admits the Square divisor cordial labeling, Sum divisor cordial labeling, Subtract divisor cordial labeling, and Multiply divisor cordial labeling.

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