

VAGUE ALPHA GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

Jeganeeswari. R¹, Francina Shalini. A²

¹PG Student, Department of Mathematics, Nirmala college for Women, Coimbatore, Tamil Nadu, India.

²Assistant Professor, Department of Mathematics, Nirmala college for Women, Coimbatore, Tamil Nadu, India.

DOI: <https://www.doi.org/10.58257/IJPREMS30722>

ABSTRACT

In this paper a new concept of vague closed sets called vague α generalized closed sets is introduced . We have constructed some examples which are useful in the theory of Vague α generalized closed sets in topological spaces.

Keywords: Vague topology, vague α generalized closed sets , vague α generalized open sets.

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh in 1965. On the otherhand Gau and Buehrer.D.J in 1993 introduced an extension of fuzzy set theory and basic concepts of vague set theory. Biswas in 2006 gave various results in vague set. The theory of fuzzy topology was introduced by Chang in 1968. Mariapresenti and Arockia Rani in 2016, defined the vague generalized alpha closed set in topological spaces. Closed sets are fundamental objects in a topological space. One can define the topology on a set by using either the axioms for the closed sets. In 1970, Levine initiated the study of generalized closed sets. By definition, a subset S of a topological space X is called generalized closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. This notion has been studied extensively in recent years by many topologists because generalized closed sets are not only natural generalizations of closed sets.

2. METHODOLOGY

PRELIMINARIES

A vague set A in the universe of discourse X is characterized by two membership functions given by:

- A true membership function $tA : X \rightarrow [0, 1]$ and
- A false membership function $fA : X \rightarrow [0, 1]$,

where $tA(x)$ is lower bound on the grade of membership of x derived from the “evidence for x”. $fA(x)$ is a lower bound on the negation of x derived from the “evidence against x” and $tA(x) + fA(x) \leq 1$. Thus the grade of membership of x in the vague set A is bounded by a sub interval $[tA(x), 1 - fA(x)]$ of $[0, 1]$. This indicates that if the actual grade of membership $\mu(x)$, then $tA(x) \leq \mu(x) \leq fA(x)$. The vague set A is written as $A = \{ \langle x, [tA(x), 1 - fA(x)] \rangle \mid x \in X \}$,

where the interval $[tA(x), 1 - fA(x)]$ is called the “vague value of x” in A and is denoted by $VA(x)$.

2.1 Definition

Let A and B be two vague sets of the form

$$A = \{ \langle x, [tA(x), 1 - fA(x)] \rangle \mid x \in X \} \text{ and } B = \{ \langle x, [tB(x), 1 - fB(x)] \rangle \mid x \in X \}.$$

Then,

- $A \cap B = \{ \langle x, [tA(x) \wedge tB(x), ((1 - fA(x) \wedge 1 - fB(x)))] \rangle \mid x \in X \}$
- $A \cup B = \{ \langle x, [tA(x) \vee tB(x), ((1 - fA(x) \vee 1 - fB(x)))] \rangle \mid x \in X \}$
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- $A^c = \{ \langle x, [tA(x), 1 - fA(x)] \rangle \mid x \in X \}$
- $A \subseteq B$ if and only if $tA \leq tB$ and $1 - fA(x) \leq 1 - fB(x)$

We shall use the notation $A = \{ \langle x, [tA(x), 1 - fA(x)] \rangle \mid x \in X \}$ instead of $\{ \langle x, [tA(x), 1 - fA(x)] \rangle \mid x \in X \}$.

2.2 Definition A vague topology (VT in short) on X is a family τ of vague sets (VS in short) in X satisfying the following axioms.

- $0, 1 \in \tau$
- $G_1 \cap G_2 \in \tau$
- $\bigcup G_i \in \tau$ for any family $\{ G_i \mid i \in J \} \subseteq \tau$

In this case the pair (X, τ) is called vague topological space (VTS in short) and vague set in τ is known as vague open

set (VOS in short) in X . The complement A^c of vague open set in vague topological space is (X, τ) called vague closed set (VCS in short) in X .

3. ANALYSIS

VAGUE α GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

Definition 3.1

A vague set A in a vague topological space X is said to be a vague α Generalized closed set (VagCS) if $V\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a vague open set (VOS) in X . The complement $C(A)$ of a VagCS A is a VagOS in X .

Theorem 3.2

Let (X, τ) be a vague topological space $A \in VagC(X)$ and for every $B \in VS(X)$, $A \subseteq B \subseteq V\alpha Cl(A)$ implies $B \in VagC(X)$.

Proof.

Let a vague set $B \subseteq U$ and U be a vague open set in (X, τ) . Then since $A \subseteq B$, $A \subseteq U$ and A is a VagCS, $V\alpha cl(A) \subseteq U$.

By hypothesis, $B \subseteq V\alpha cl(A)$.

Therefore $V\alpha cl(B) \subseteq V\alpha cl(V\alpha cl(A)) \subseteq U$, Therefore, $V\alpha cl(B) \subseteq U$.

Hence B is a VagCS(X).

A vague set A in a vague topological space X is said to be a vague α Generalized closed set (VagCS) if $V\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a vague open set (VOS) in X . The complement $C(A)$ of a VagCS A is a VagOS in X .

Theorem 3.3

The union of any two VagCSs is a VagCS in a vague topological space X .

Proof.

Let A and B be any two VagCSs in a vague topological space X . Let $A \subseteq B \subseteq U$ where U is a vague open set in X .

Then $A \subseteq U$ and $B \subseteq U$.

Now $V\alpha cl(A \cup B) = (A \cup B) \subseteq Vcl(VInt(Vcl(A \cup B))) \subseteq (A \cup B) \subseteq Vcl(Vcl(A$

$\cup B)) \subseteq (A \cup B) \subseteq Vcl(A \cup B) \subseteq Vcl(A \cup B) = Vcl(A) \cup Vcl(B) \subseteq U \subseteq U = U$, by hypothesis. Hence $A \cup B$ is a VagCS in X .

Theorem 3.4

Every Vague α closed set A is a VagCS in X but not converse.

Proof.

Let $A \subseteq U$ where U is a Vague open set in X .

Now $V\alpha cl(A) = A \subseteq Vcl(VInt(Vcl(A))) \subseteq A \subseteq A = A \subseteq U$ by hypothesis. Therefore, A is a VagCS in X .

Theorem 3.5

Every Vague α closed set A is $V\alpha CS$ in X but not converse.

Proof.

Let $A \subseteq U$, U is a Vague open set in X .

Then, $V\alpha Cl(A) = A \subseteq Vcl(VInt(Vcl(A))) \subseteq A \subseteq A = A \subseteq U$, by hypothesis. Hence A is a $V\alpha CS$ in X .

Theorem 3.6

If A is a vague open set and a VagCS in (X, τ) . Then A is a $V\alpha$ -CS in (X, τ) .

Proof.

Since $A \subseteq A$ and

A is a Vague open set in (X, τ) . By hypothesis, $V\alpha cl(A) \subseteq A$.

But $A \subseteq V\alpha Cl(A)$. Therefore $V\alpha Cl(A) = A$.

Hence A is a $V\alpha$ -CS in (X, τ) .

4. CONCLUSION

This dissertation is a study of Vague α generalized closed sets in topological spaces. The Vague α generalized closed

sets and Vague α generalized open sets in topological spaces and further defined some examples related to open and closed sets .

5. REFERENCES

- [1] Biswas. R, Vague groups, Internat J Comput. Cognition. 2006; 4[2]:20-23.
- [2] Bustince. H, Burillo. P, Vague Sets are Intuitionistic Fuzzy Sets, Fuzzy sets and systems, 79, (1996), 403-405.
- [3] Chang. C.L, Fuzzy Topological Spaces, Math. J Analysis appl.24, 182-190, (1968).
- [4] Coker, D, An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy sets and systems, 88, 1997, 81-89.
- [5] Dontchev. J, On generating semi- preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A Math. 16 (1995) 35-48.
- [6] El-Shafhi , M.E., and Zhakari, A., Semi generalized continuous mappings in fuzzy topological spaces, J. Egypt. Math. Soc. 15 [1] [2007], 57-67 .
- [7] Ganster.M and Reilly I.I, Locally closed sets LC continuous functions, Internat.J.Math, Ma
- [8] Gau.W.L, Buehrer. D. J, Vague Sets, IEEE Trans, Systems Man and Cyber- net, 23[2], [1993], 610-614.
- [9] Ishwarya. P and Bageerathi. K. On Neutrosophic Semi Open Sets in Neutrosophic Topological Spaces, International Jour. of Math. Trends and Tech. 2016, 214-223.