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VAGUE FEEBLY CLOSED SETS & VAGUE FEEBLY OPEN SETS IN VAGUE TOPOLOGICAL SPACES

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ABSTRACT

This paper studies the concepts of a new class of Vague Feebly Closed sets & Vague Feebly Open sets in vague topological space also some basic properties and the key theorems of these classes were discussed here.

Keywords- Vague set (VS), Vague topology (VT), Vague Feebly closed set (V$\mathcal{F}$CS), Vague Feebly open set (V$\mathcal{F}$OS).

1. INTRODUCTION

The theory of vague sets was first initiated by Gau and Buehrer [1] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. Maheswari and Jain (1978) [4], Ibraheem (2008) [2,3] introduced feebly open and feebly closed sets, feebly generalized closed (briefly $\mathcal{F}$g-closed) sets, generalized feebly closed (briefly g$\mathcal{F}$ -closed) sets respectively. In 2017,Vigneshwaran.M and Velmeenal. M [5], studied on R$\mathcal{F}$G -closed sets in topological spaces.

In this article, we introduce the concept of Vague feebly open sets and Vague feebly closed sets in VTS. We also analyzed their characterizations and investigated their properties with suitable examples. For a subset A of a VTS (X, τ), vague feebly closure of A, vague feebly interior of A and the vague complement of A are denoted by $V\mathcal{F}cl(A), V\mathcal{F}int(A)$ and $V(A^C)$ respectively.

1.1 Vague Feebly Open And Vague Feebly Closed Sets

Definition 1.1.1: Let A and B be any two vague subsets of a VTS. Then A is vague q-neighbourhood with B if there exists a VOS O with AqO ⊆ B. If A is not vague quasi-coincident with B then we write $A_{\neg}qB$.

Thus $A_{\neg}qB$ if and only if for each $x \in X$, $A(x) \subseteq B^c(x)$. i.e., $A \subset B^c$.

Proposition 1.1.2: Let (X, τ) be a VTS. Then for a VS A of a VTS X, Vscl(A) is the union of all vague points $Vx_{(\alpha,\beta)}$ such that every vague semi open set O with $Vx_{(\alpha,\beta)}qO$ is vague q-coincident with A.

Proof: Let $x_i \in Vscl(A)$.

Suppose there is a vague semi - open set 'O' such that $Vx_{(\alpha,\beta)}qO$ and $O_{\neg q}A$.

$\Longrightarrow O^c \supseteq A$, where O^c is vague semi - closed.

Also, $O^c \supseteq Vscl(A)$ and $Vx_{(\alpha,\beta)} \notin O^c$

$\Longrightarrow Vx_{(\alpha,\beta)} \notin Vscl(A)$. This is a contradiction to our assumption.

Therefore, for every vague semi - open set 'O' with $Vx_{(\alpha,\beta)}qO$ is vague q-coincident with A.

Conversely, for every vague semi - open set 'O' with $Vx_{(\alpha,\beta)}qO$ is vague q-coincident with A. Suppose $x_i \in Vscl(A)$. Then there is a vague semi - closed set $G \supseteq A$ with$Vx_{(\alpha,\beta)} \notin G$. Hence $V(G^c)$ is a vague semi - open set with $Vx_{(\alpha,\beta)}q(G^c)$ and $G^c{}_{\neg}qA$. i.e., $A(x) \supset (G^c)^c = G$. This is a contradiction to our assumption. Therefore, $Vx_{(\alpha,\beta)} \in Vscl(A)$.

Proposition 1.1.3: Let (X, τ) be a VTS. Let A and B are two vague subsets of a VTS. Then

- $A_{\neg}qB \Leftrightarrow A \subseteq B^c$.
- If $A \cap B = 0_v$ then $A_{\neg}qB$
- $A \subseteq B \Leftrightarrow Vx_{(\alpha,\beta)}qB$, for each $Vx_{(\alpha,\beta)}qA$.

Proof: (i) Proof follows from the definition 1.1.1

(ii) Let $(A \cap B)(x) = 0_v$. Then $\min\{A(x), B(x)\} = 0_v$

$\Longrightarrow A(x) = 0_v$ and $B(x) = 1_v$ (or) $B(x) = 0_v$ and $A(x) = 1_v$

(i.e) $B^c \supseteq (1_v)^c = A$ (or) $A^c \supseteq (1_v)^c = B$

$\Longrightarrow A \subseteq B^c$.

Hence $A_{\neg}qB$. This proves (ii).

(iii) Let $A \subseteq B$ and $Vx_{(\alpha,\beta)}qA$. Then $A^c(x) \subseteq (Vx_{(\alpha,\beta)^c})$

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Also $A \subseteq B$ implies that $A^c \supseteq B^c \Longrightarrow B^c \subseteq Vx_{(\alpha,\beta)^c}$.

i.e., $Vx_{(\alpha,\beta)} \subseteq B$.

Therefore, $Vx_{(\alpha,\beta)} qB$. Thus each $Vx_{(\alpha,\beta)} qA$, $Vx_{(\alpha,\beta)} qB$.

Suppose, $A(x) \supset B(x)$. Then $A^c \subseteq Vx_{(\alpha,\beta)}$ does not implies $B^c \subseteq {Vx_{(\alpha,\beta)}}^c$

This is a contradiction to our assumption.

Therefore $A(x) \subseteq B(x)$. This proves (iii).

Proposition 1.1.4: Let (X, τ) be a VTS. Let A be a vague subset of a VTS (X, τ). Then

- $Vint(Vcl(Vint(Vcl(A)))) = Vint(Vcl(A))$ and
- $Vcl(Vint(Vcl(Vint(A)))) = Vcl(Vint(A))$
- $(Vint(Vcl(A)))^c = Vcl(Vint(A^c))$ and $(Vcl(Vint(A)))^c = Vint(Vcl(A^c))$.

Proof: (i) It is true that.

$Vint(Vcl(A)) \subseteq Vcl(A)$

$Vcl\left(Vint(Vcl(A))\right) \subseteq Vcl(Vcl(A)) = Vcl(A)$

$\Longrightarrow Vint(Vcl(Vint(Vcl(A)))) \subseteq Vint(Vcl(A))$

Since $Vint(Vcl(A))$ is vague open, $Vint(Vcl(A)) \subseteq Vcl\left(Vint(Vcl(A))\right)$,

$Vint(Vcl(A)) = Vint(Vint(Vcl(A))) \subseteq Vint(Vcl(Vint(Vcl(A))))$

From the above, we have

$Vint(Vcl(Vint(Vcl(A)))) = Vint(Vcl(A))$. This proves (i).

(ii) It is true that, $Vint(A^c) = (Vcl(A))^c$ and $Vcl(A^c) = (Vint(A))^c$

By this (ii) is proved.

Proposition: 1.1.5: Let (X, τ) be a VTS. Let A be a vague subset of a VTS (X, τ). Then $Vint(Vcl(A)) \subseteq Vscl(A)$.

Proof: Let $Vx_{(\alpha,\beta)} \in Vint(Vcl(A))$.

Then by using the proposition 1.1.2, $Vx_{(\alpha,\beta)} \subseteq Vint(Vcl(A)(x))$.

This implies that $x_{(\alpha,\beta)} \in Vscl(A)$.

i.e., $Vint(Vcl(A)) \subseteq Vscl(A)$.

Theorem 1.1.6: Let (X, τ) be a VTS. If a vague subset A is vague open, then $Vint(Vcl(A)) = Vscl(A)$.

Proof: By using the above proposition 1.1.5, we have $Vint(Vcl(A)) \subseteq Vscl(A)$.

Therefore it is sufficient to prove $Vscl(A) \subseteq Vint(Vcl(A))$.

Let $Vx_{(\alpha,\beta)} \notin Vint(Vcl(A))$. Then $Vx_{(\alpha,\beta)} q(Vint(Vcl(A)))^c$.

By using proposition 1.1.2, $x_{(\alpha,\beta)} q(Vcl(Vint(A^c)))$.

By using proposition 1.1.4, $Vcl(Vint(A^c)) = Vcl(Vint(Vcl(Vint(A^c))))$

This can be written as $Vcl(Vint(A^c)) \subseteq Vcl(Vint(Vcl(Vint(A)^c)))$.

Also, $Vcl(Vint(A^c))$ is vague semi - open. By using proposition 4.1.3, we have

$A_{\urcorner q} Vcl(Vint(A)^c)$

$\Longrightarrow Vx_{(\alpha,\beta)} \notin Vscl(A)$

$\Longrightarrow Vscl(A) \subseteq Vint(Vcl(A))$

Therefore $Vint(Vcl(A)) = Vscl(A)$.

Theorem 1.1.7: Let (X, τ) be a VTS. If a vague subset A is vague closed,

then $Vcl(Vint(A)) = Vsint(A)$.

Proof: If A is vague closed, then $V(A^c)$ is vague open.

By theorem 1.1.6,

$Vint(Vcl(A^c)) \subseteq Vscl(A^c)$.

Then by $(Vcl(Vint(A)))^C \subseteq (Vsint(A))^C$.

Taking complement on both sides, we get $Vcl(Vint(A)) \subseteq Vsint(A)$.

Definition 1.1.8: A subset A in a VTSX is called Vague feebly open in X if there exists an VOS U such that $U \subseteq A \subseteq Vscl(U)$. The complement of $V\mathcal{F}OS$ is a $V\mathcal{F}CS$.

Proposition 1.1.9: A vague subset A of a VTS (X, τ) is $V\mathcal{F}OS$ if and only if $A \subseteq Vint(Vcl(Vint(A)))$.

Proof: If A is $V\mathcal{F}OS$, then by the definition 1.1.8, we have $U \subseteq A \subseteq Vscl(U)$, where U is a VOS. Then by theorem 1.1.6, $U \subseteq A \subseteq Vint(Vcl(U))$.

Since U is vague open, we have $U = Vint(U) \subseteq Vint(A)$,

it follows that $Vcl(U) \subseteq Vcl\big(Vint(A)\big)$

$\Longrightarrow Vint(Vcl(U)) \subseteq Vint(Vcl(Vint((A)))$.

Thus, $A \subseteq Vint(Vcl(U)) \subseteq Vint(Vcl(Vint((A)))$.

Assume that $\subseteq Vint(Vcl(Vint((A)))$. Now, $Vint(A) \subseteq A$.

$\Longrightarrow Vint(A) \subseteq Vint(Vcl(Vint((A)))$. Take $= Vint(A)$.

Then U is a VOS in X, such that $U \subseteq A \subseteq Vint(Vcl(U))$.

By theorem 1.1.6, $U \subseteq A \subseteq Vscl(U)$.

Therefore, A is $V\mathcal{F}OS$.

Theorem 1.1.10: Let (X, τ) be a VTS. A set A is said to be a $V\mathcal{F}OS$ if and only if $A \subseteq Vscl(Vint(A))$.

Proof : Follows from proposition 1.1.5 and proposition 1.1.9.

The following example is an example of $V\mathcal{F}OS$.

Example 1.1.11: Let $X = \{a, b\}, \tau = \{0, 1, G\}$, where $G = \{< x, [0.4, 0.7], [0.2, 0.4] >\}$

Let $A = \{< x, [0.4, 0.7], [0.2, 0.4] >\}$. Here $A \subseteq Vint(Vcl(Vint((A))) = 1$.

Hence A is a $V\mathcal{F}OS$.

Definition 4.1.12: A vague subset A of a VTS (X, τ) is a $V\mathcal{F}OS$ if $A \subseteq Vscl(Vint(A))$ and $V\mathcal{F}CS$ if $Vsint(Vcl(A)) \subseteq A$.

Proposition 1.1.13: Every VOS is a $V\mathcal{F}OS$.

Proof: Let A be a VOS in X.

Therefore $A = Vint(A)$ and $A \subseteq Vcl(Vint(A))$.

Now, $A \subseteq Vint(Vcl(Vint((A)))$.

$\Longrightarrow A \subseteq Vint(Vcl(Vint((A)))$.

Hence A is a vague feebly open set.

The converse of the above proposition is not true as shown in the example below.

Example 1.1.14: Let $X = \{a, b\}, \tau = \{0, 1, G\}$,

where $G = \{< x, [0.4, 0.7], [0.2, 0.4] >\}$ then (X, τ) be a VTS.

Let $= \{< x, [0.4, 0.7], [0.2, 0.4] >\}$.

Here is not a VOS since $Vint(A) \neq A$.

But $Vint(Vcl(Vint((A))) = 1$.

Hence, $A \subseteq Vint(Vcl(Vint((A)))$.

Therefore, A is $V\mathcal{F}OS$.

Proposition 1.1.15: A vague subset A in a VTS is a $V\mathcal{F}OS$ if and only if it is vague semi - open and vague pre - open.

Proof: Let A be a $V\mathcal{F}OS$ in X.

Then $A \subseteq Vint(Vcl(Vint((A)))$

$\Longrightarrow A \subseteq Vint(Vcl(Vint((A))) \subseteq Vcl(Vint((A))$.

Hence A is a vague semi - open set.

Since A is $V\mathcal{F}OS$ in X, we have

$A \subseteq Vint(Vcl(Vint((A)))$

$\Longrightarrow A \subseteq Vint(Vcl(Vint((A))) \subseteq Vint(Vcl\big((A)\big))$.

Hence A is vague pre - open set.

Conversely, let A is a vague semi - open set,

Therefore $A \subseteq Vcl(Vint((A))$ so that $(Vcl(A)) \subseteq Vcl(Vcl(Vint((A)))$.

Hence, $Vint(Vcl((A))) \subseteq Vint(Vcl(Vint(A)))$.

Since A is a vague pre - open set, $A \subseteq Vint(Vcl((A))$ and

hence $A \subseteq Vint(Vcl(Vint(A))$.

Then by proposition 1.1.9**,** A is $V\mathcal{F}OS$.

Definition 1.1.16: Let (X, τ) be a VTS and $A \subseteq X$.

- The intersection of all Vague feebly closed subsets of the space X containing A is called the Vague feebly closure of A and denoted by $V\mathcal{F}cl(A)$ and also
- $V\mathcal{F}cl(A) = A \cup Vsint(Vcl(A))$.
- The union of all Vague feebly open subsets of the space X contained in A is called Vague feebly interior of A and is denoted by $V\mathcal{F}int(A)$.

It is known that $V\mathcal{F}int(A) = A \cap Vscl(Vint(A))$.

Proposition 1.1.17 : If A and B are two $V\mathcal{F}OS$ then $A \cup B$ is a $V\mathcal{F}OS$.

Proof: If A and B are two $V\mathcal{F}OS$, then by proposition 1.1.9**,**

$A \subseteq Vint(Vcl(Vint(A)))$ and $B \subseteq Vint(Vcl(Vint(B)))$.

Now $A \cup B \subseteq Vint(Vcl(Vint(A))) \cup Vint(Vcl(Vint(B)))$.

Since $t(A) \cup Vint(B) \subseteq Vint(A \cup B)$,

$A \cup B \subseteq Vint(Vcl(Vint(A))) \cup Vcl(Vint(B)))$

Also, $A \cup B \subseteq Vint(Vcl(Vint(A))) \cup Vint(B)$

This implies $\cup B \subseteq Vint(Vcl(Vint(A \cup B)))$.

Hence $A \cup B$ is a $V\mathcal{F}OS$.

Proposition 1.1.18: Arbitrary union of vague feebly open sets is a vague feebly open set.

Proof: Let $\{A_i\}$ be a collection of $V\mathcal{F}OSs$ of a VTS (X, τ).

Then there exists a VOS U_i such that

$Ui \subseteq Ai \subseteq Vscl(Ui)$ for each i.

Now $\cup Ui \subseteq \cup Ai \subseteq \cup Vscl(Ui)$

$\Rightarrow \cup Ui \subseteq \cup Ai \subseteq Vscl(\cup Ui)$.

Hence $\cup A_i$ is a $V\mathcal{F}OS$.

Example 1.1.19: Intersection of any two $V\mathcal{F}OSs$ need not be a $V\mathcal{F}OS$ as shown in the example below.

Let $X = \{a, b\}, \tau = \{0, 1, G_1, G_2, G_3, G_4\}$ be a vague topology on X.

where $G_1 = \{< x, [0.5, 0.8], [0.5, 0.6] >\}, G_2 = \{< x, [0.4, 0.5], [0.6, 0.7] >\}$,

$G_3 = G_1 \cup G_2 = \{< x, [0.5, 0.8], [0.4, 0.7] >\}$ and

$G_4 = G_1 \cap G_2 = \{< x, [0.4, 0.5], [0.4, 0.6] >\}$. Let $A = \{< x, [0.5, 0.8], [0.4, 0.7] >\}$ and $B = \{< x, [0.2, 0.5], [0.5, 0.4] >\}$ be $V\mathcal{F}OSs$ in (X, τ)

but $A \cap B = \{< x, [0.2, 0.5], [0.4, 0.4] >\}$ is not a $V\mathcal{F}OS$ in (X, τ).

Proposition 1.1.20: The vague closure of a VOS is a $V\mathcal{F}OS$.

Proof: Let A be a VOS in X.

Take $A = Vint(A)$,

Now, $Vcl(A) = Vcl(Vint(A))$.

Since $A \subseteq Vcl(A)$.

$Vint(A) \subseteq Vint(Vcl(A))$.

$A \subseteq Vint(Vcl(Vint(A)))$.

Hence A is a $V\mathcal{F}OS$.

Proposition 1.1.21: Let A be a $V\mathcal{F}OS$ in the VTS (X, τ) and suppose $A \subseteq B \subseteq Vscl(A)$.

Then B is a $V\mathcal{F}OS$.

Proof: Let A be a $V\mathcal{F}OS$ in the VTS (X,τ) and

B be any vague subset of X such that $A \subseteq B \subseteq Vscl(A)$.

Since A is $V\mathcal{F}OS$, there exists a VOS U such that $U \subseteq A \subseteq Vscl(U)$.

Since $U \subseteq B$ and $Vscl(A) \subseteq Vscl(U)$ and thus $B \subseteq Vscl(A) \Longrightarrow U \subseteq B \subseteq Vscl(U)$.

Hence B is $V\mathcal{F}OS$.

Definition 1.1.22: A vague subset A of (X,τ) is said to be a vague feebly generalised closed set ($V\mathcal{F}GCS$ in short) if $V\mathcal{F}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $V\mathcal{F}OS$ in X.

Definition 1.1.23: A vague subset A of (X,τ) is said to be a vague generalised feebly closed set ($VG\mathcal{F}CS$ in short) if $V\mathcal{F}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a VOS in X.

Definition 1.1.24: A vague subset A of a VTS (X,τ) is $V\mathcal{F}CS$ if there is a VCS U in X such that $Vsint(U) \subseteq A \subseteq U$.

Proposition 1.1.25: A vague subset A of a VTS (X,τ) is $V\mathcal{F}CS$ if and only if $Vcl(Vint(Vcl(A) \subseteq A$.

Proof: If A is $V\mathcal{F}CS$ then by the definition 1.1.24

there is a VCS U such that $Vsint(U) \subseteq A \subseteq U$. Also $Vcl\big(Vint(U)\big) \subseteq A \subseteq U$.

Since U is a VCS, $Vcl(A) \subseteq U = Vcl(U)$.

Therefore $Vcl(Vint\,(Vcl(A))) \subseteq Vcl(Vint(U)) \subseteq A$.

Hence $Vcl\left(Vint\,(Vcl(A))\right) \subseteq A$.

Conversely, Assume that $Vcl(Vint(Vcl(A))) \subseteq A$.

Since $Vcl(A) \supseteq A$, $Vcl(A) \supseteq Vcl\left(Vint(Vcl(A))\right)$. Take $U = Vcl(A)$.

Then U is a VCS in X such that $Vsint(U) \subseteq A \subseteq U$.

By the definition 1.1.24, A is a $V\mathcal{F}CS$.

Proposition 1.1.26: A vague subset A is a $V\mathcal{F}CS$ if and only if $V(A^c)$ is a $V\mathcal{F}OS$.

Proof: Let A be a $V\mathcal{F}CS$.

Then by the proposition 1.1.2, $Vcl(Vint(Vcl(A))) \subseteq A$.

Taking compliment on both sides $Vcl(Vint(Vcl(A)))^c \supseteq A^c$.

This implies $A^c \subseteq Vcl(Vint(Vcl(A^c)))$.

Hence A^c is a $V\mathcal{F}OS$.

Conversely, let A^c is a $V\mathcal{F}OS$, then $A^c \subseteq Vint(Vcl(Vint(A^c)))$.

Taking complement on both sides, $(A^c)^c \supseteq (Vint\left(Vcl(Vint(A^c))\right))^c$.

Then $A \supseteq Vcl(Vint(Vcl(A)))$.

Therefore $Vcl(Vint(Vcl(A))) \subseteq A$.

Hence A is a $V\mathcal{F}CS$.

Theorem 1.1.27: A vague subset A is a $V\mathcal{F}CS$if and only if $Vsint\,(Vcl(A)) \subseteq A$.

Proof: Let A be a $V\mathcal{F}CS$. Then A^c is vague feebly open.

By using theorem 1.1.10 $A^c \subseteq Vscl(Vint(A^c))$.

Taking complement on both sides $(Vsint\,(Vcl(A)))^c \supseteq A^c$. $A^c \subseteq Vsint(Vcl(A^c))$. Therefore A^c is a $V\mathcal{F}OS$. By proposition 1.1.26. A is a $V\mathcal{F}CS$.

The following is an example of $V\mathcal{F}CS$.

Example 1.1.28: Let $X = \{a, b\}, \tau = \{0, 1, G\}$, where $G = \{< x, [0.4, 0.7], [0.2, 0.4] >\}$

then (X,τ) be a VTS and let $A = \{< x, [0.3, 0.6], [0.6, 0.8] >\}$.

Here $Vint(Vcl(Vint((A))) \subseteq A$.

Therefore, $A = \{< x, [0.3, 0.6], [0.6, 0.8] >\}$ is a $V\mathcal{F}CS$.

Proposition 1.1.29: Every VCS is $V\mathcal{F}CS$.

Proof: Let A be a VCS in X. Then $A = Vcl(A)$.

Since $Vint\,(A) \subseteq A, Vint\big(Vcl(A)\big) \subseteq A \Rightarrow Vcl\left(Vint\big(Vcl(A)\big)\right) \subseteq Vcl(A) = A$.

By proposition 1.1.25, A is a $V\mathcal{F}CS$.

The converse of the above theorem need not be true as shown in the example below

Example 1.1.30: Let $X = \{a, b\}, \tau = \{0, 1, G\}$,

where $G = \{< x, [0.4, 0.7], [0.2, 0.4] >\}$ then (X, τ) be a vague topological space and

let $A = \{< x, [0.2, 0.6], [0.6, 0.7] >\}$.

Here A is a $V\mathcal{F}CS$.

Proposition 1.1.31: If A and B are any two $V\mathcal{F}CSs$, then $Vsint(Vcl(A)) \subseteq A$ and $Vsint\,(Vcl(B)) \subseteq B$.

Proof: By theorem 1.1.27, $Vsint\,\big(Vcl(A)\big) \cap Vsint\,\big(Vcl(B)\big) \subseteq A \cap B$.

This implies $Vsint\,\big(Vcl(A)\big) \cap Vcl(B)) \subseteq A \cap B$. T

his implies $Vsint(Vcl(A \cap B)) \subseteq A \cap B$.

Hence $A \cap B$ is a $V\mathcal{F}CS$.

Proposition 1.1.32: Finite intersection of $V\mathcal{F}CSs$ is a $V\mathcal{F}CS$.

Proof: Let$\{A_i\}$ be a collection of $V\mathcal{F}CSs$ of a VTS (X, τ).

Then by the definition 1.1.24 there exists a VCS V_i such that

$V_i sint(V_i) \subseteq A_i \subseteq V_i$ for each i.

Now $\cap V_i sint(V_i) \subseteq \cap A_i \subseteq \cap V_i$

$\Longrightarrow Vsint(\cap V_i) \subseteq \cap A_i \subseteq \cap V_i$

Hence $\cap A_i$ is a $V\mathcal{F}CS$.

Remark 1.1.32: Union of any two $V\mathcal{F}CSs$ need not be a $V\mathcal{F}CS$as shown in the example.

Example 1.1.33: Let $X = \{a, b\}, \tau = \{0, 1, G_1, G_2, G_3, G_4\}$ be a VT on X,

where $G_1 = \{< x, [0.5, 0.8], [0.5, 0.6] >\}, G_2 = \{< x, [0.4, 0.5], [0.6, 0.7] >\}$,

$G_3 = G_1 \cup G_2 = \{< x, [0.5, 0.8], [0.4, 0.7] >\}$ and

$G_4 = G_1 \cap G_2 = \{< x, [0.4, 0.5], [0.4, 0.6] >\}$ and

let a VS $A = \{< x, [0.2, 0.5], [0.3, 0.6] >\}$ and $B = \{< x, [0.5, 0.8], [0.6, 0.5] >\}$ be two $V\mathcal{F}CSs$ in (X, τ) but $A \cup B = \{< x, [0.5, 0.8], [0.6, 0.6] >\}$ is not a $V\mathcal{F}CS$ in (X, τ).

2. CONCLUSION

This article has delved into the intricate concepts of vague feebly closed sets and vague feebly open sets within the framework of vague topological spaces. Through a rigorous exploration of these mathematical constructs, we have unveiled their properties and relationships and characteristics of vague topology. As we wrap up this study, it becomes evident that the investigation of vague feebly closed and open sets opens avenues for further research and exploration. We propose the following areas for future work: Extension to Higher Dimensions, Relation to Other Topological Concepts, Applications in Real-world Problems and Generalization to Different Vague Topological Spaces.

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