

## MATHEMATICAL MODEL THE HEAT PUMP DRYER VARIABLES EQUATIONS & APPROACH

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### ABSTRACT

To develop a Model for the performance of the basic components of the Vapor compression wet sheet dryers (compressor, condenser, evaporator , expansion device and the wet sheet Dryer ) could be done to optimize the design of the finned-tube heat exchangers ( evaporator or the condenser) in this kind of application.

### 1. INTRODUCTION

Heat pump drying is the technology that invokes to efficiently use the energy. Heat pump, by its name, is a device implicitly supplying heat mainly in space heating application and heat recovery. Heat pump application in drying has continuously received attention since it possesses two-fold beneficial characteristic. Through the evaporator, the heat pump recuperates sensible and latent heat from the dryer exhaust, hence the energy is recovered. Condensation occurring at the evaporator reduces the humidity of the working air, thus increases the driving force for product drying. It is, therefore, anticipated that the heat pump dryer (HPD) can accelerate the drying process and use energy efficiently. Furthermore, heat pump dryer is suitable for temperature-sensitive products because effective drying can occur at low temperature (because of low humidity).

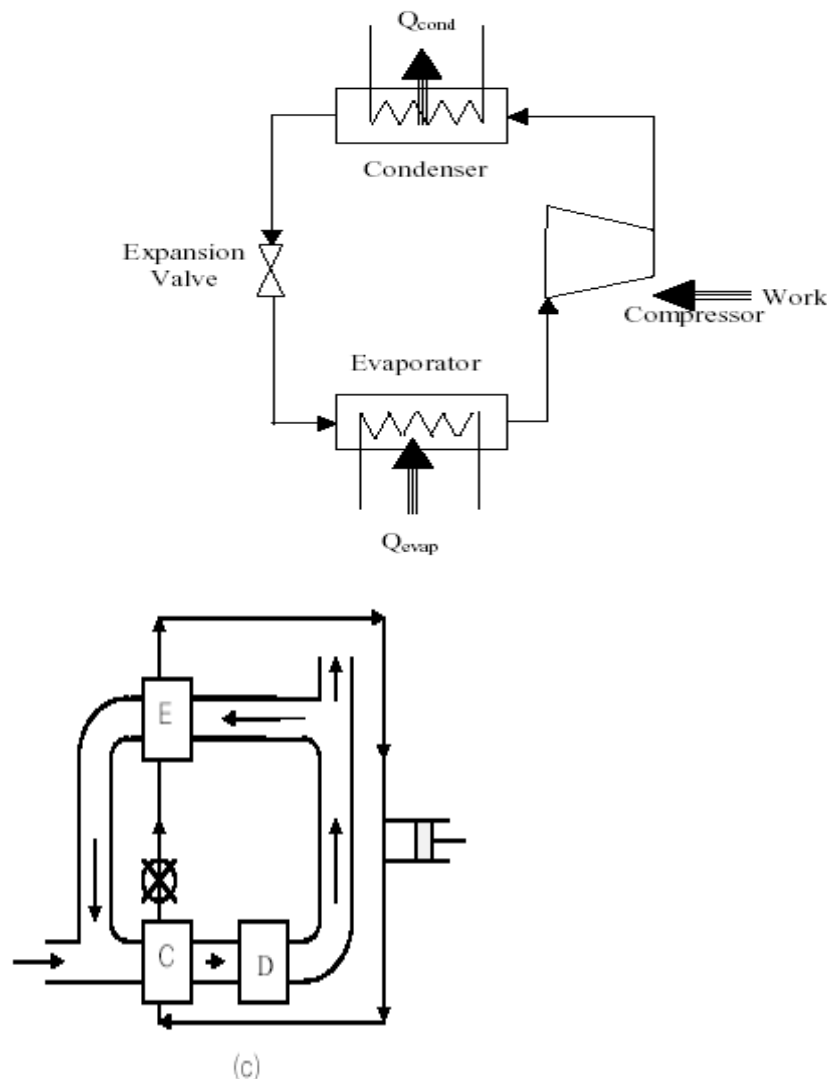


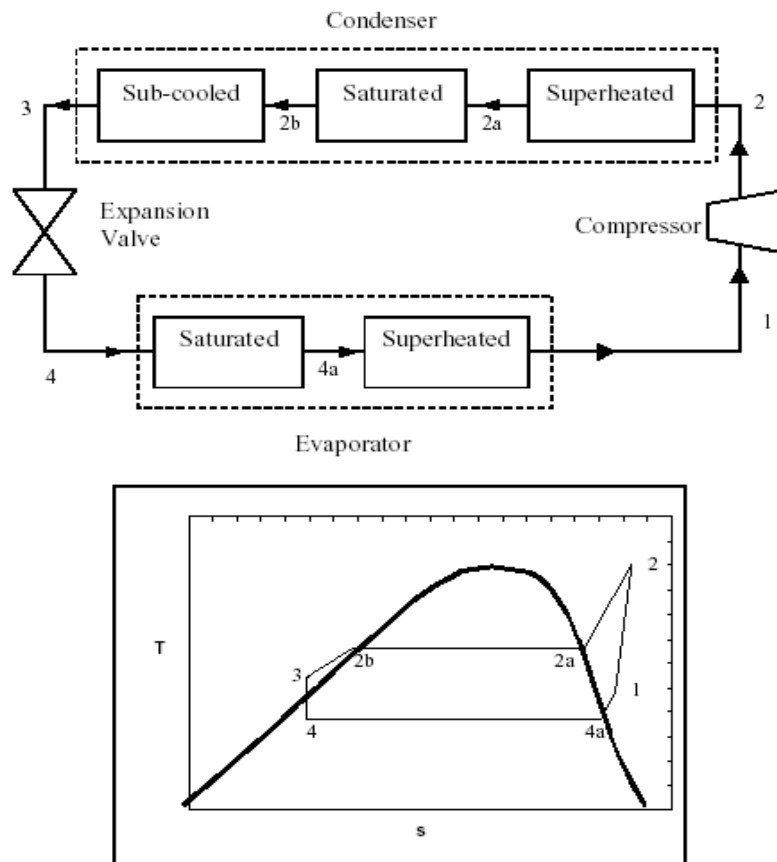
Figure 1. Heat pump dryer models (E = evaporator , C = condenser , D = dryer)

The superheat, saturated, and subcool portions of the heat exchanger will be modeled separately and in detail using appropriate pressure drop and heat transfer fundamental equations for both the air-side and refrigerant-side of the heat exchangers. The study will use accurate refrigerants and air properties and can be modified from a refrigerant to other as a characteristic of the EES software.

The compressor model uses the physical indicative volumetric efficiency equation and the constant polytropic exponent and a catalog data fitted Clearance volume ratio , a suction pressure drop and a adiabatic efficiency mode .

The Cycle cooling output and electrical input to evaporator and condenser fans Will be also calculated for various ambient temperature conditions. The condenser fan, and evaporator components of the cycle are also modeled but in a more global manner using thermal science laws.

The cycle coefficient of performance will be the optimization objective function.



## 2. METHODOLOGY

### The compressor Model :

The model used in this study will be based on the volumetric efficiency which is defined as the mass of vapor that is actually pumped divided by the mass of vapor that the compressor could pump if it could handle the total piston displacement at the suction state ( Ref to the below indicator diagram ) .

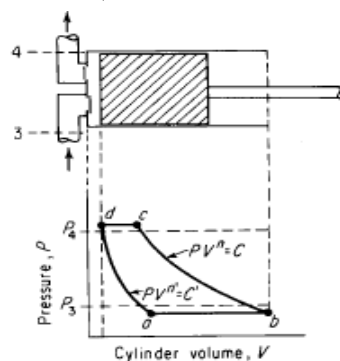


Figure 2-2: Schematic indicator diagram for a reciprocating compressor [Threlkeld, 1962]

From the above indicator diagram shows the refrigerant states that are being used in the following derivation. The volumetric efficiency can be represented by:

$$\eta_v = \frac{V_b - V_a}{V_b - V_d} = \frac{(V_b - V_a) \cdot v_{suction}}{(V_b - V_d) \cdot v_b} \quad (3-1)$$

where  $\eta_v$  - volumetric efficiency  
 $V_b$  - total displacement volume  
 $V_a$  - volume of the re-expanded clearance vapor  
 $V_d$  - clearance volume  
 $v_b$  - specific volume of the refrigerant in the cylinder after the intake (re-expanded clearance vapor mixed with fresh intake)  
 $v_{suction}$  - specific volume of the refrigerant at suction line conditions

The compression and expansion process are described as polytropic processes. It is assumed that they have the same polytropic exponent  $n$ .

$$p_{suction} \cdot V_a^n = p_{discharge} \cdot V_d^n$$

with  $p_{suction}$  - suction pressure  
 $p_{discharge}$  - discharge pressure  
 $n$  - polytropic exponent

$$\frac{V_a}{V_d} = \left( \frac{p_{discharge}}{p_{suction}} \right)^{\frac{1}{n}}$$

The clearance volume ratio  $C$  can be expressed as

$$C = \frac{V_d}{V_b - V_d}$$

and accordingly

$$\eta_v = \left[ 1 + C - C \left( \frac{p_{discharge}}{p_{suction}} \right)^{\frac{1}{n}} \right] \cdot \frac{v_{suction}}{v_b}$$

and as

$$\eta_v = \frac{\dot{m} \cdot v_{suction}}{V \cdot RPM}$$

Finally we get this equation for the refrigerant mass flow rate.

$$\dot{m} = \left[ 1 + C - C \left( \frac{p_{discharge}}{p_{suction}} \right)^{\frac{1}{n}} \right] \cdot \frac{V \cdot RPM}{v_b}$$

we are going to fit the compressor catalog data using above volumetric efficiency equation with a constant polytropic exponent. And with two parameters Clearance volume ratio  $C$  and suction pressure drop  $p_{suction}$  such that:

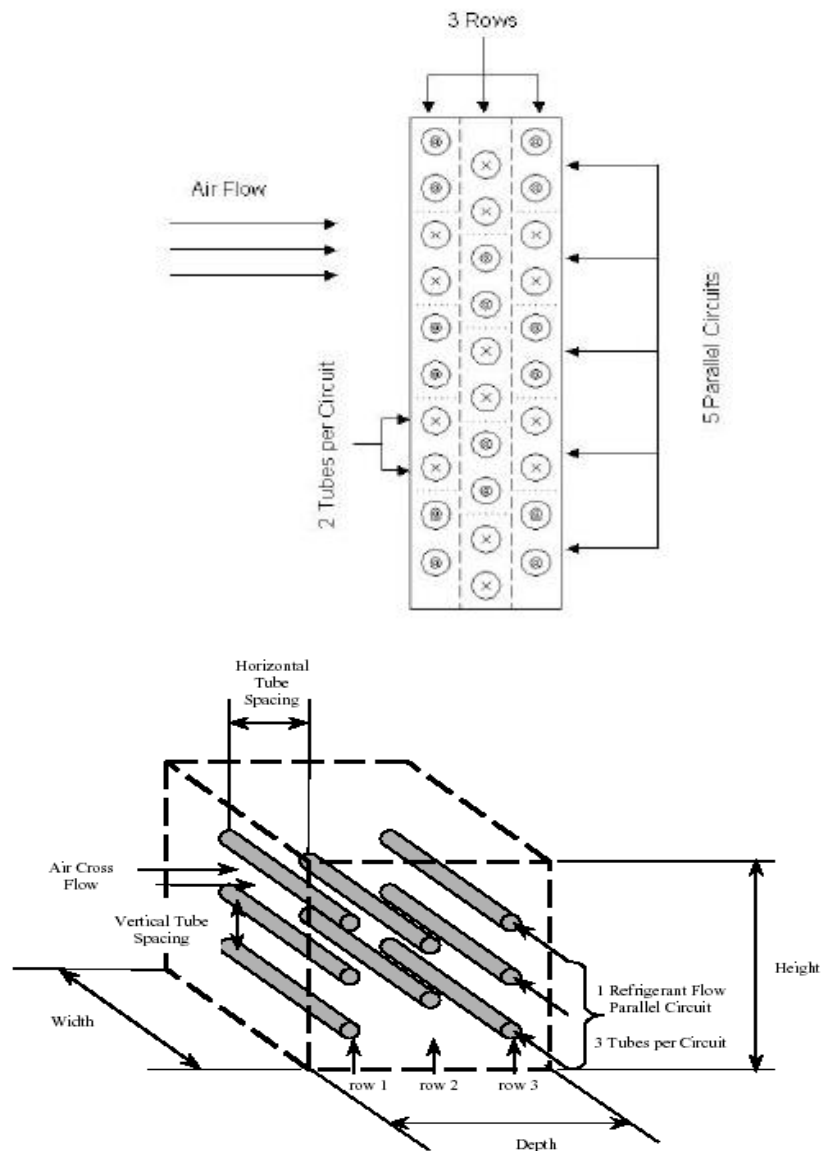
$$\dot{m}_{calc} = \left[ 1 + C - C \left( \frac{p_{cond}}{p_{evap}(1 - \Delta p)} \right)^{\frac{1}{n}} \right] \cdot \frac{V \cdot RPM}{v_{suction} \cdot 60}$$



### The Evaporator and the condenser Models

The following Figures represent the Geometric composition of both of the Condenser and the evaporator :

- The term “tubes per circuit” is the number of parallel passages the refrigerant mass flow rate is divided among.
- The number of parallel circuits is used to determine the number of tubes in each row. If the number of parallel circuits is set to 12, and the number of tubes per circuit is 2, then there will be a total of 24 tubes in each row.
- The number of rows refers to the number of tube rows in the direction normal to air flow.



### NTU-Effectiveness Relations

For the heat exchanger, the total heat rejected from the hot fluid, in this case, refrigerant, to the cold fluid, air, is dependent on the heat exchanger effectiveness and the heat capacity of each fluid.:

$$Q = \varepsilon C_{\min} (T_{h,i} - T_{c,i})$$

As in both of the condenser and the evaporator there is more than a refrigerant state then the following applies for each portion

$$\frac{m_{a,sat}}{m_{a,tot}} = \frac{L_{sat}}{L_{tot}}$$

The equations used to determine the effectiveness depend on the temperature

distribution within each fluid and on the paths of the fluids as heat transfer takes place, ie. parallel-flow, counter-flow or cross-flow. In typical condensers and evaporators, the refrigerant mass flow is separated into a number of tubes and does not mix. As the air flows through the fins, the plates prevent mixing and air at one end of the heat exchanger will not necessarily be the same temperature as air at the other end. For a cross-flow heat exchanger with both fluids unmixed, the effectiveness can be related to the number of transfer units (NTU) with the following equation:

$$\varepsilon = 1 - \exp \left[ \left( \frac{1}{C_r} \right) (NTU)^{0.22} \left\{ \exp \left[ -C_r (NTU)^{0.78} \right] - 1 \right\} \right]$$

Cr = heat capacity ratio

$$C_r = \frac{C_{\min}}{C_{\max}}$$

In the saturated portion of the condenser, the heat capacity on the refrigerant side approaches infinity and the heat capacity ratio goes to zero. When Cr =0, the effectiveness is calculated with the following equation :

$$\varepsilon = 1 - \exp(-NTU)$$

Where

$$NTU = \frac{UA}{C_{\min}}$$

And

$$\frac{1}{UA} = \frac{1}{\eta_{s,a} \bar{h}_a A_a} + \frac{R_{f,a}''}{\eta_{s,a} A_a} + R_w + \frac{R_{f,r}''}{\eta_{s,r} A_r} + \frac{1}{\eta_{s,r} \bar{h}_r A_r}$$

The surface Efficiency

$$\eta_s = 1 - \frac{A_f}{A_o} (1 - \eta_f)$$

where:

$\eta_s$  = surface efficiency

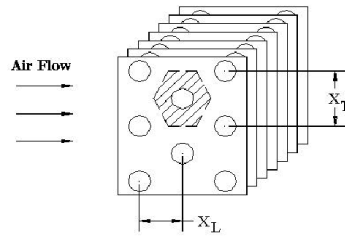
Af = total fin surface area

Ao = total air side surface area, tube and fins

$\eta_f$  = fin efficiency

$$\eta_f = \frac{\tanh(mr_e \phi)}{(mr_e \phi)}$$

For a plate fin heat exchanger with multiple rows of staggered tubes, the plates can be evenly divided into hexagonal shaped fins as shown in Figure



$$\frac{r_e}{r} = 1.27\psi(\beta - 0.3)^{1/2}$$

where:

$r_e$  = equivalent radius of fins

$r$  = outside tube radius

The coefficients  $\psi$  and  $\beta$  are defined as:

$$\psi = \frac{X_t}{2r}$$

$$\beta = \frac{1}{X_t} \left( X_L^2 + \frac{X_t^2}{4} \right)^{1/2}$$

Once the equivalent radius has been determined, the equations for standard circular fins can be used. For the fins in this study, the length is much greater than the thickness, so a parameter  $m$  can be expressed as:

$$m = \left( \frac{2\bar{h}_a}{kt} \right)^{1/2}$$

where:  $h_a$  = air side heat transfer coefficient

$k$  = conductivity of fin material

$t$  = thickness of fins

And

$$\phi = \left( \frac{r_e}{r} - 1 \right) \left[ 1 + 0.35 \ln \left( \frac{r_e}{r} \right) \right]$$

#### The Heat transfer for the air side of the condenser (without condensation)

The model is developed for dry coils. The heat transfer coefficient is based on the Colburn  $j$ -factor, which is defined as:

$$j = St Pr^{2/3}$$

Substituting the appropriate values for the **Stanton number**,  $St$ , gives the following relationship for the air-side convective heat transfer coefficient,  $h_a$ ,

$$\bar{h}_a = \frac{j c_p G_{\max}}{Pr^{2/3}}$$

where

cp is the specific heat,

Gmax is the mass flux of air through the minimum flow area which is expressed as:

$$G_{\max} = \frac{\dot{m}_{\text{air}}}{A_{\min}}$$

**McQuiston** use a 4-row finned tube heat exchanger as the baseline model, and define the Colburn j-factor for a 4-row finned-tube heat exchanger as:

$$j_4 = 0.2675JP + 1.325 \times 10^{-6}$$

$$JP = \text{Re}_D^{-0.4} \left( \frac{A_o}{A_t} \right)^{-0.15}$$

Ao is the total air side heat transfer surface area (fin area plus tube area), and At is the tube outside surface area. The Reynolds number, ReD in the above expression is based on the outside diameter of the tubes, Do, and the maximum mass flux, Gmax. The area ratio can be expressed as:

$$\frac{A_o}{A_t} = \frac{4}{\pi} \frac{X_l}{D_h} \frac{X_t}{D_{\text{depc}}} \sigma$$

Ddepc is the depth of the condenser in the direction of the air flow, Dh is the hydraulic

$$D_h = \frac{4 A_{\min} D_{\text{depc}}}{A_o}$$

$$\sigma = \frac{A_{\min}}{A_{\text{fr}}}$$

The j-factor for heat exchangers with four or fewer rows can be found using the following correlation, where z is No. of rows, and Rers is based on the row spacing, Xrs,

$$\frac{j_z}{j_4} = \frac{1 - 1280z \text{Re}_{rs}^{-1.2}}{1 - (1280)(4) \text{Re}_{rs}^{-1.2}}$$

$$\text{Re}_{rs} = \frac{G_{\max} X_{rs}}{\mu}$$

### 3. REFRIGERANT SIDE MODELS

#### Single Phase Heat Transfer Coefficient

To find the single phase heat transfer coefficient, the standard heat transfer equations and the experimental work of Kays and London were considered. Kays and London have established equations in the transition region. The heat transfer coefficient was related to the Stanton number, St. The Stanton number is defined by the following:

$$St = \frac{h}{G c_p}$$

$$St \text{ Pr}^{2/3} = a \text{ Re}^b$$



where:  $c_p$  = specific heat

The coefficients a and b are based on the flow regime.

Laminar	$Re < 3,500$	$a=1.10647$ $b=-0.78992$
Transition	$3,500 < Re < 6,000$	$a=3.5194 \times 10^{-7}$ $b=1.03804$
Turbulent	$6,000 < Re$	$a=0.2243$ $b=-0.385$

The heat transfer Coefficient For complete condensation,

$$h_{TPM} = h_L \left( 0.55 + \frac{2.09}{P_r^{0.38}} \right)$$

Evaporative heat transfer Coefficient

$$\bar{h}_{evap} = (0.0186875) \frac{k_l}{D^{0.2}} \left( \frac{G}{\mu_l} \right)^{0.8} \left( \frac{\mu_l C_{p,l}}{k_l} \right)^{0.4} \left( \frac{\rho_l}{\rho_v} \right)^{0.375} \left( \frac{\mu_v}{\mu_l} \right)^{0.075} \left( \frac{x_e - x_l}{x_e^{0.325} - x_l^{0.325}} \right)$$

Pressure drop,

Single phase pressure drop equations are, Straight tube pressure drop,

$$\Delta P = \frac{\tau L \rho V^2}{2 D_i}$$

where,

$$\tau = \frac{64}{Re} \quad \text{for } Re < 2300$$

$$\frac{1}{\tau^{0.5}} = 2.0 \log(Re \tau^{0.5}) - 0.8 \quad \text{for } Re > 2300$$

pressure drop in bending and friction coefficient was defined as

$$\tau = 144 \frac{(5.58 \times 10^{-6})(Re_v^{0.5})}{\exp\left(\frac{0.215 C_d}{D_i}\right) X^{1.25}}$$

## Two phase pressure drop equations in Straight tube pressure drop,

$$\Delta P = \int_0^L \left( \frac{dP}{dZ} \right) dZ$$

For horizontal tube, the components of the total pressure gradient are related to wall friction and acceleration gradient as follows:

$$\left( \frac{dP}{dZ} \right) = \left( \frac{dP}{dZ} \right)_f + \left( \frac{dP}{dZ} \right)_a$$

where,

$$\left( \frac{dP}{dZ} \right)_f = -0.09 \left( \frac{G_v^2}{\rho_v D_i} \right) \left( \frac{\mu_v}{G_v D_i} \right)^{0.2} \left\{ 1 - 2.85 \left[ \left( \frac{\mu_l}{\mu_v} \right)^{0.1} \left( \frac{1-X}{X} \right)^{0.9} \left( \frac{\rho_v}{\rho_l} \right)^{0.5} \right]^{0.523} \right\}^2$$

$$\left( \frac{dP}{dZ} \right)_a = - \left( \frac{G_v^2}{\rho_v} \right) \left( \frac{dX}{dZ} \right) \left[ 2X + (1-2X) \left( \frac{\rho_v}{\rho_l} \right)^{1/3} + (1-2X) \left( \frac{\rho_v}{\rho_l} \right)^{2/3} - 2(1-X) \left( \frac{\rho_v}{\rho_l} \right) \right]$$

and

$$\frac{dX}{dZ} = \frac{X_o - X_i}{L}$$

$$G_v = GX$$

## Expansion Device

For TXV the Evaporator load is calculated from Maps as follows

$$\dot{m}_{r, \text{rated}} = \frac{12,000 \dot{Q}_{\text{rated}} b_{\text{fac}}}{h_{\text{evap out, rated}} - h_{\text{liquid line, rated}}}$$

Where  $Q_{\text{rated}}$  is function in degree of sub cool and TXV type and evaporation temperature as follows

**THERMOSTATIC EXPANSION VALVE CAPACITIES for REFRIGERANTS TONS OF REFRIGERATION**

**AIR CONDITIONING, HEAT PUMP and**

VALVE TYPES	NOMINAL CAPACITY	REFRIGERANT					
		22					
		RECOMMENDED THERMOSTATIC VALVE TYPES					
		VC	VCP100	VGA	VZ	VZPH	
		EVAPORATOR TEMPERATURE					
		40°	20°	0°	-10°	-20°	
F-EF-Q-EG	1/8	0.20	0.22	0.19	0.17	0.11	
N	1/4	0.25	0.27	0.25	0.22	0.11	
F-EF-Q-EG	1/2	0.35	0.38	0.33	0.27	0.2	
N-F-EF-Q-EG	3/4	0.45	0.49	0.40	0.35	0.3	
Q-EG	1	0.75	0.80	0.71	0.60	0.6	
N-F-EF-Q-EG	1	1.00	1.09	0.95	0.88	0.7	

Table A

The valve capacity should equal or slightly exceed the tonnage rating of the system. (For complete R-22 capacity tables, see page 6)

Design  
Evaporating  
Temperature

**Table B**

REFRIGERANT	LIQUID TEMPERATURE EN							
	0°	10°	20°	30°	40°	50°	60°	70°
22	1.56	1.51	1.45	1.40	1.34	1.29	1.23	1.17
407A	1.75	1.68	1.61	1.53	1.46	1.39	1.31	1.21
407C	1.69	1.62	1.55	1.49	1.42	1.35	1.28	1.21

Liquid Temperature

Table B

**Table C**

EVAPORATOR TEMPERATURE (°F)	PRESSURE DROP ACROSS TEV (PSI)					
	30	50	75	100	125	150
40°	0.55	0.71	0.87	1.00	1.12	1.22
20° & 0°	0.49	0.63	0.77	0.89	1.00	1.10
-10° & -20°	0.45	0.58	0.71	0.82	0.91	1.00
-40°	0.41	0.53	0.65	0.76	0.85	0.93

TEV Pressure Drop

#### Selection Example – Refrigerant 22

Application: medium temperature refrigeration	
Design evaporator temperature	20°F
Design condenser temperature	95°F
Refrigerant liquid temperature	70°F
Design system capacity	1 ton

Available pressure drop across TEV:	
Condensing pressure (psig)	182
Evaporating pressure (psig)	43
	139
Liquid line and accessories loss (psi)	4
Distributor and tubes loss (psi) ①	35
	100

Refrigerant liquid correction factor	1.17
Pressure drop correction factor	0.89

Use the following formula to calculate TEV capacity:  
TEV Capacity = TEV rating x CF liquid temperature x CF pressure drop

EGVE-1 Has valve capacity of:  $1.09 \times 1.17 \times 0.89 = 1.13$  Tons  
at 20°F evaporating temperature, 100 psi pressure drop and  
70° liquid temperature.

#### 4. CONCLUSION

The collection of the Equations required to Model the Heat Pump Dryer is a challenge and the Equations were collected and indicated as per the paper details this forms the first part in creating a numerical Model to implement the design parameters and study their effect on the cycle

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