

INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS) e-ISSN : 2583-1062

www.ijprems.com editor@ijprems.com

Vol. 04, Issue 02, February 2024, pp: 561-566

Impact Factor: 5.725

# VAGUE FEEBLY CLOSED SETS & VAGUE FEEBLY OPEN SETS IN VAGUE TOPOLOGICAL SPACES

# Mary Tency E. L<sup>1</sup>

<sup>1</sup>Assistant Professor, Department of Mathematics Sri Ramakrishna College of Arts & Science,

Coimbatore – 06, India.

DOI: https://www.doi.org/10.58257/IJPREMS32711

#### ABSTRACT

This paper studies the concepts of a new class of Vague Feebly Closed sets & Vague Feebly Open sets in vague topological space also some basic properties and the key theorems of these classes were discussed here.

Keywords- Vague set (VS), Vague topology (VT), Vague Feebly closed set (VFCS), Vague Feebly open set (VFOS).

#### 1. INTRODUCTION

The theory of vague sets was first initiated by Gau and Buehrer [1] as an extension of fuzzy set theory and vague sets are regarded as a special case of context-dependent fuzzy sets. Maheswari and Jain (1978) [4], Ibraheem (2008) [2,3] introduced feebly open and feebly closed sets, feebly generalized closed (briefly  $\mathcal{F}g$ -closed) sets, generalized feebly closed (briefly  $\mathcal{F}\mathcal{F}$ -closed) sets respectively. In 2017, Vigneshwaran.M and Velmeenal. M [5], studied on R $\mathcal{F}G$  - closed sets in topological spaces.

In this article, we introduce the concept of Vague feebly open sets and Vague feebly closed sets in VTS. We also analyzed their characterizations and investigated their properties with suitable examples. For a subset A of a VTS  $(X, \tau)$ , vague feebly closure of A, vague feebly interior of A and the vague complement of A are denoted by  $V\mathcal{F}cl(A)$ ,  $V\mathcal{F}int(A)$  and  $V(A^{C})$  respectively.

#### 1.1 Vague Feebly Open And Vague Feebly Closed Sets

**Definition 1.1.1:** Let A and B be any two vague subsets of a VTS. Then A is vague q-neighbourhood with B if there exists a VOS 0 with AqO  $\subseteq$  B. If A is not vague quasi-coincident with B then we write A<sub>7</sub>qB.

Thus  $A_7 qB$  if and only if for each  $x \in X$ ,  $A(x) \subseteq B^c(x)$ . i.e.,  $A \subset B^c$ .

**Proposition 1.1.2:** Let  $(X, \tau)$  be a VTS. Then for a VS A of a VTS X, Vscl(A) is the union of all vague points  $Vx_{(\alpha,\beta)}$  such that every vague semi open set O with  $Vx_{(\alpha,\beta)}qO$  is vague q-coincident with A.

**Proof:** Let  $x_i \in Vscl(A)$ .

Suppose there is a vague semi - open set 'O' such that  $Vx_{(\alpha,\beta)}qO$  and  $O_{7q}A$ .

 $\Rightarrow$  O<sup>c</sup>  $\supseteq$  A, where O<sup>c</sup> is vague semi - closed.

Also,  $O^c \supseteq Vscl(A)$  and  $Vx_{(\alpha,\beta)} \notin O^c$ 

 $\Rightarrow$  Vx<sub>( $\alpha,\beta$ )</sub>  $\notin$  Vscl(A). This is a contradiction to our assumption.

Therefore, for every vague semi - open set 'O' with  $Vx_{(\alpha,\beta)}qO$  is vague q-coincident with A.

Conversely, for every vague semi - open set 'O' with  $Vx_{(\alpha,\beta)}qO$  is vague q-coincident with A. Suppose  $x_i \in Vscl(A)$ . Then there is a vague semi - closed set  $G \supseteq A$  with  $Vx_{(\alpha,\beta)} \notin G$ . Hence  $V(G^c)$  is a vague semi - open set with  $Vx_{(\alpha,\beta)}q(G^c)$  and  $G^c_{\gamma}qA$ . i.e.,  $A(x) \supset (G^c)^c = G$ . This is a contradiction to our assumption. Therefore,  $Vx_{(\alpha,\beta)} \in Vscl(A)$ .

**Proposition 1.1.3:** Let  $(X, \tau)$  be a VTS. Let A and B are two vague subsets of a VTS. Then

- $A_7qB \Leftrightarrow A \subseteq B^c$ .
- If  $A \cap B = 0_v$  then  $A_7 qB$
- $A \subseteq B \Leftrightarrow Vx_{(\alpha,\beta)}qB$ , for each  $Vx_{(\alpha,\beta)}qA$ .

**Proof:** (i) Proof follows from the definition 1.1.1

(ii) Let  $(A \cap B)(x) = 0_v$ . Then min $\{A(x), B(x)\} = 0_v$ 

 $\Rightarrow$  A(x) = 0<sub>v</sub> and B(x) = 1<sub>v</sub> (or) B(x) = 0<sub>v</sub> and A(x) = 1<sub>v</sub>

(i.e) 
$$B^c \supseteq (1_v)^c = A$$
 (or)  $A^c \supseteq (1_v)^c = B$ 

$$\Rightarrow A \subseteq B^{c}.$$

Hence A7qB. This proves (ii).

(iii) Let  $A \subseteq B$  and  $Vx_{(\alpha,\beta)}qA$ . Then  $A^{c}(x) \subseteq (Vx_{(\alpha,\beta)}c)$ 



## INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)

www.ijprems.com Vol. 04, Issue 02, February 2024, pp: 561-566 5.725 editor@ijprems.com Also  $A \subseteq B$  implies that  $A^c \supseteq B^c \Longrightarrow B^c \subseteq Vx_{(\alpha,\beta)^c}$ . i.e.,  $Vx_{(\alpha,\beta)} \subseteq B$ . Therefore,  $Vx_{(\alpha,\beta)}qB$ . Thus each  $Vx_{(\alpha,\beta)}qA$ ,  $Vx_{(\alpha,\beta)}qB$ . Suppose,  $A(x) \supset B(x)$ . Then  $A^c \subseteq Vx_{(\alpha,\beta)}$  does not implies  $B^c \subseteq Vx_{(\alpha,\beta)}^c$ This is a contradiction to our assumption. Therefore  $A(x) \subseteq B(x)$ . This proves (iii). **Proposition 1.1.4:** Let  $(X, \tau)$  be a VTS. Let A be a vague subset of a VTS  $(X, \tau)$ . Then Vint(Vcl(Vint(Vcl(A)))) = Vint(Vcl(A)) and • Vcl(Vint(Vcl(Vint(A)))) = Vcl(Vint(A))  $(Vint(Vcl(A)))^{c} = Vcl(Vint(A^{c}))$  and  $(Vcl(Vint(A)))^{c} = Vint(Vcl(A^{c}))$ . **Proof:** (i) It is true that.  $Vint(Vcl(A)) \subseteq Vcl(A)$  $Vcl(Vint(Vcl(A))) \subseteq Vcl(Vcl(A)) = Vcl(A)$  $\Rightarrow$  Vint(Vcl(Vint(Vcl(A))))  $\subseteq$  Vint(Vcl(A)) Since Vint(Vcl(A)) is vague open,  $Vint(Vcl(A)) \subseteq Vcl(Vint(Vcl(A)))$ ,  $Vint(Vcl(A)) = Vint(Vint(Vcl(A))) \subseteq Vint(Vcl(Vint(Vcl(A))))$ From the above, we have Vint(Vcl(Vint(Vcl(A)))) = Vint(Vcl(A)). This proves (i). (ii) It is true that,  $Vint(A^c) = (Vcl(A))^c$  and  $Vcl(A^c) = (Vint(A))^c$ By this (ii) is proved. **Proposition:** 1.1.5: Let  $(X, \tau)$  be a VTS. Let A be a vague subset of a VTS  $(X, \tau)$ . Then  $Vint(Vcl(A)) \subseteq Vscl(A)$ . **Proof:** Let  $Vx_{(\alpha,\beta)} \in Vint(Vcl(A))$ . Then by using the proposition 1.1.2,  $Vx_{(\alpha,\beta)} \subseteq Vint(Vcl(A)(x))$ . This implies that  $x_{(\alpha,\beta)} \in Vscl(A)$ . i.e.,  $Vint(Vcl(A)) \subseteq Vscl(A)$ . **Theorem 1.1.6:** Let  $(X, \tau)$  be a VTS. If a vague subset A is vague open, then Vint(Vcl(A)) = Vscl(A). **Proof:** By using the above proposition 1.1.5, we have  $Vint(Vcl(A)) \subseteq Vscl(A)$ . Therefore it is sufficient to prove  $Vscl(A) \subseteq Vint(Vcl(A))$ . Let  $Vx_{(\alpha,\beta)} \notin Vint(Vcl(A))$ . Then  $Vx_{(\alpha,\beta)}q(Vint(Vcl(A)))^c$ . By using proposition 1.1.2,  $x_{(\alpha,\beta)}q(Vcl(Vint(A^c)))$ . By using proposition 1.1.4,  $Vcl(Vint(A^c)) = Vcl(Vint(Vcl(Vint(A^c))))$ This can be written as  $Vcl(Vint(A^c)) \subseteq Vcl(Vint(Vcl(Vint(A)^c)))$ . Also,  $Vcl(Vint(A^{c}))$  is vague semi - open. By using proposition 4.1.3, we have  $A_{7q}Vcl(Vint(A)^{c})$  $\Rightarrow$  Vx<sub>( $\alpha,\beta$ )</sub>  $\notin$  Vscl(A)  $\Rightarrow$  Vscl(A)  $\subseteq$  Vint(Vcl(A)) Therefore Vint(Vcl(A)) = Vscl(A). **Theorem 1.1.7:** Let  $(X, \tau)$  be a VTS. If a vague subset A is vague closed, then Vcl(Vint(A)) = Vsint(A). **Proof:** If A is vague closed, then  $V(A^c)$  is vague open. By theorem 1.1.6,  $Vint(Vcl(A^c)) \subseteq Vscl(A^c).$ Then by  $(Vcl(Vint(A)))^{C} \subseteq (Vsint(A))^{C}$ . Taking complement on both sides, we get  $Vcl(Vint(A)) \subseteq Vsint(A)$ .



#### e-ISSN: INTERNATIONAL JOURNAL OF PROGRESSIVE 2583-1062 **RESEARCH IN ENGINEERING MANAGEMENT** Impact AND SCIENCE (IJPREMS)

Vol. 04, Issue 02, February 2024, pp: 561-566

**Factor:** 5.725

www.ijprems.com editor@ijprems.com

**Definition 1.1.8:** A subset A in a VTSX is called Vague feebly open in X if there exists an VOS U such that  $U \subseteq A \subseteq$ Vscl(U). The complement of VFOS is a VFCS. **Proposition 1.1.9:** A vague subset A of a VTS  $(X, \tau)$  is VFOS if and only if  $A \subseteq Vint(Vcl(Vint(A)))$ . **Proof:** If A is VFOS, then by the definition 1.1.8, we have  $U \subseteq A \subseteq Vscl(U)$ , where U is a VOS. Then by theorem 1.1.6,  $U \subseteq A \subseteq Vint(Vcl(U))$ . Since U is vague open, we have  $U = Vint(U) \subseteq Vint(A)$ , it follows that  $Vcl(U) \subseteq Vcl(Vint(A))$  $\Rightarrow$  Vint(Vcl(U))  $\subseteq$  Vint(Vcl(Vint((A))). Thus,  $A \subseteq Vint(Vcl(U)) \subseteq Vint(Vcl(Vint((A))))$ . Assume that  $\subseteq$  Vint(Vcl(Vint((A))). Now, Vint(A)  $\subseteq$  A.  $\Rightarrow$  Vint(A)  $\subseteq$  Vint(Vcl(Vint((A))). Take = Vint(A). Then U is a VOS in X, such that  $U \subseteq A \subseteq Vint(Vcl(U))$ . By theorem 1.1.6,  $U \subseteq A \subseteq Vscl(U)$ . Therefore, A is VFOS. **Theorem 1.1.10:** Let  $(X, \tau)$  be a VTS. A set A is said to be a VFOS if and only if  $A \subseteq Vscl(Vint(A))$ . **Proof**: Follows from proposition 1.1.5 and proposition 1.1.9. The following example is an example of VFOS. **Example 1.1.11:** Let  $X = \{a, b\}, \tau = \{0, 1, G\}$ , where  $G = \{\langle x, [0.4, 0.7], [0.2, 0.4] \rangle \}$ Let  $A = \{ \langle x, [0.4, 0.7], [0.2, 0.4] \rangle \}$ . Here  $A \subseteq Vint(Vcl(Vint((A))) = 1$ . Hence A is a VFOS. **Definition 4.1.12:** A vague subset A of a VTS  $(X, \tau)$  is a VFOS if A  $\subseteq$  Vscl(Vint(A)) and VFCS if Vsint(Vcl(A))  $\subseteq$ A. **Proposition 1.1.13:** Every VOS is a VFOS. Proof: Let A be a VOS in X. Therefore A = Vint(A) and  $A \subseteq Vcl(Vint(A))$ . Now,  $A \subseteq Vint(Vcl(Vint((A))))$ .  $\Rightarrow$  A  $\subseteq$  Vint(Vcl(Vint((A))). Hence A is a vague feebly open set. The converse of the above proposition is not true as shown in the example below. **Example 1.1.14:** Let  $X = \{a, b\}, \tau = \{0, 1, G\}, \tau = \{0, 1, G\},$ where  $G = \{ \langle x, [0.4, 0.7], [0.2, 0.4] \rangle \}$  then  $(X, \tau)$  be a VTS. Let  $= \{ < x, [0.4, 0.7], [0.2, 0.4] > \}.$ Here is not a VOS since  $Vint(A) \neq A$ . But Vint(Vcl(Vint((A))) = 1. Hence,  $A \subseteq Vint(Vcl(Vint((A))))$ . Therefore, A is VFOS. **Proposition 1.1.15:** A vague subset A in a VTS is a VFOS if and only if it is vague semi - open and vague pre - open. **Proof:** Let A be a  $V\mathcal{F}OS$  in X. Then  $A \subseteq Vint(Vcl(Vint((A))))$  $\Rightarrow$  A  $\subseteq$  Vint(Vcl(Vint((A)))  $\subseteq$  Vcl(Vint((A)). Hence A is a vague semi - open set. Since A is  $V\mathcal{F}OS$  in X, we have

 $A \subseteq Vint(Vcl(Vint((A))))$ 

 $\Rightarrow A \subseteq Vint(Vcl(Vint((A))) \subseteq Vint(Vcl((A))).$ 



www.ijprems.com

#### INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)

Impact Easter

Vol. 04, Issue 02, February 2024, pp: 561-566

Factor: 5.725

#### editor@ijprems.com Hence A is vague pre - open set.

Conversely, let A is a vague semi - open set,

Therefore  $A \subseteq Vcl(Vint((A))$  so that  $(Vcl(A)) \subseteq Vcl(Vcl(Vint((A))))$ .

Hence,  $Vint(Vcl((A))) \subseteq Vint(Vcl(Vint(A)))$ .

Since A is a vague pre - open set,  $A \subseteq Vint(Vcl((A)))$  and

hence  $A \subseteq Vint(Vcl(Vint(A)))$ .

Then by proposition 1.1.9, A is  $V\mathcal{F}OS$ .

#### **Definition 1.1.16:** Let $(X, \tau)$ be a VTS and $A \subseteq X$ .

- The intersection of all Vague feebly closed subsets of the space X containing A is called the Vague feebly closure of A and denoted by VFcl(A) and also
- $V\mathcal{F}cl(A) = A \cup Vsint(Vcl(A)).$
- The union of all Vague feebly open subsets of the space X contained in A is called Vague feebly interior of A and is denoted by VFint(A).

It is known that  $V\mathcal{F}int(A) = A \cap Vscl(Vint(A))$ .

**Proposition 1.1.17 :** If A and B are two VFOS then A  $\cup$  B is a VFOS.

**Proof:** If A and B are two VFOS, then by proposition 1.1.9,

 $A \subseteq Vint(Vcl(Vint(A)))$  and  $B \subseteq Vint(Vcl(Vint(B)))$ .

Now  $A \cup B \subseteq Vint(Vcl(Vint(A))) \cup Vint(Vcl(Vint(B)))$ .

Since  $t(A) \cup Vint(B) \subseteq Vint(A \cup B)$ ,

 $A \cup B \subseteq Vint(Vcl(Vint(A))) \cup Vcl(Vint(B)))$ 

Also,  $A \cup B \subseteq Vint(Vcl(Vint(A))) \cup Vint(B)$ 

This implies  $\bigcup B \subseteq Vint(Vcl(Vint(AUB)))$ .

Hence AUB is a VFOS.

Proposition 1.1.18: Arbitrary union of vague feebly open sets is a vague feebly open set.

**Proof:** Let  $\{A_i\}$  be a collection of VFOSs of a VTS  $(X, \tau)$ .

Then there exists a VOS U<sub>i</sub> such that

 $Ui \subseteq Ai \subseteq Vscl(Ui)$  for each i.

Now  $\cup$  Ui  $\subseteq \cup$  Ai  $\subseteq \cup$  Vscl(Ui)

 $\Rightarrow \cup Ui \subseteq \cup Ai \subseteq Vscl(\cup Ui).$ 

Hence  $\cup A_i$  is a VFOS.

**Example 1.1.19:** Intersection of any two VFOSs need not be a VFOS as shown in the example below.

Let  $X = \{a, b\}, \tau = \{0, 1, G_1, G_2, G_3, G_4\}$  be a vague topology on X.

where  $G_1 = \{ < x, [0.5, 0.8], [0.5, 0.6] > \}, G_2 = \{ < x, [0.4, 0.5], [0.6, 0.7] > \},$ 

 $G_3 = G_1 \cup G_2 = \{ < x, [0.5, 0.8], [0.4, 0.7] \}$ and

 $G_4 = G_1 \cap G_2 = \{ < x, [0.4, 0.5], [0.4, 0.6] > \}$ 

 $B = \{ \langle x, [0.2, 0.5], [0.5, 0.4] \rangle \} \text{ be } V\mathcal{F}OSs \text{ in } (X, \tau)$ 

Let  $A = \{ < x, [0.5, 0.8], [0.4, 0.7] \}$  and

but  $A \cap B = \{ \langle x, [0.2, 0.5], [0.4, 0.4] \rangle \}$  is not a VFOS in  $(X, \tau)$ .

**Proposition 1.1.20:** The vague closure of a VOS is a VFOS.

**Proof:** Let A be a VOS in X.

Take A = Vint(A),

Now, Vcl(A) = Vcl(Vint(A)).

Since  $A \subseteq Vcl(A)$ .

 $Vint(A) \subseteq Vint(Vcl(A)).$ 

 $A \subseteq Vint(Vcl(Vint(A))).$ 

Hence A is a VFOS.

**Proposition 1.1.21:** Let A be a VFOS in the VTS  $(X, \tau)$  and suppose A  $\subseteq$  B $\subseteq$  Vscl(A).

Then B is a  $V\mathcal{F}OS$ .



## INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)

e-ISSN : 2583-1062

Impact **Factor:** www.ijprems.com Vol. 04, Issue 02, February 2024, pp: 561-566 5.725 editor@ijprems.com **Proof:** Let A be a VFOS in the VTS  $(X, \tau)$  and B be any vague subset of X such that  $A \subseteq B \subseteq Vscl(A)$ . Since A is VFOS, there exists a VOS U such that  $U \subseteq A \subseteq Vscl(U)$ . Since  $U \subseteq B$  and  $Vscl(A) \subseteq Vscl(U)$  and thus  $B \subseteq Vscl(A) \Longrightarrow U \subseteq B \subseteq Vscl(U)$ . Hence B is VFOS. **Definition 1.1.22:** A vague subset A of  $(X, \tau)$  is said to be a vague feebly generalised closed set (VFGCS in short) if  $V\mathcal{F}cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is a  $V\mathcal{F}OS$  in X. **Definition 1.1.23:** A vague subset A of  $(X, \tau)$  is said to be a vague generalised feebly closed set (VGFCS in short) if  $V\mathcal{F}cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is a VOS in X. **Definition 1.1.24:** A vague subset A of a VTS  $(X, \tau)$  is VFCS if there is a VCS U in X such that  $Vsint(U) \subseteq A \subseteq U$ . **Proposition 1.1.25:** A vague subset A of a VTS  $(X, \tau)$  is VFCS if and only if Vcl(Vint(Vcl(A)  $\subseteq$  A. **Proof:** If A is VFCS then by the definition 1.1.24 there is a VCS U such that  $Vsint(U) \subseteq A \subseteq U$ . Also  $Vcl(Vint(U)) \subseteq A \subseteq U$ . Since U is a VCS,  $Vcl(A) \subseteq U = Vcl(U)$ . Therefore  $Vcl(Vint(Vcl(A))) \subseteq Vcl(Vint(U)) \subseteq A$ . Hence  $Vcl(Vint(Vcl(A))) \subseteq A$ . Conversely, Assume that  $Vcl(Vint(Vcl(A))) \subseteq A$ . Since  $Vcl(A) \supseteq A$ ,  $Vcl(A) \supseteq Vcl(Vint(Vcl(A)))$ . Take U = Vcl(A). Then U is a VCS in X such that  $Vsint(U) \subseteq A \subseteq U$ . By the definition 1.1.24, A is a  $V\mathcal{F}CS$ . **Proposition 1.1.26:** A vague subset A is a V $\mathcal{F}$ CS if and only if V(A<sup>c</sup>) is a V $\mathcal{F}$ OS. **Proof:** Let A be a  $V\mathcal{F}CS$ . Then by the proposition 1.1.2,  $Vcl(Vint(Vcl(A))) \subseteq A$ . Taking compliment on both sides  $Vcl(Vint(Vcl(A))))^{c} \supseteq A^{c}$ . This implies  $A^c \subseteq Vcl(Vint(Vcl(A^c)))$ . Hence A<sup>c</sup> is a VFOS. Conversely, let  $A^c$  is a VFOS, then  $A^c \subseteq Vint(Vcl(Vint(A^c)))$ . Taking complement on both sides,  $(A^c)^c \supseteq (Vint(Vcl(Vint(A^c))))^c$ . Then  $A \supseteq Vcl(Vint(Vcl(A)))$ . Therefore  $Vcl(Vint(Vcl(A))) \subseteq A$ . Hence A is a VFCS. **Theorem 1.1.27:** A vague subset A is a V $\mathcal{F}$ CSif and only if Vsint (Vcl(A))  $\subseteq$  A. **Proof:** Let A be a V $\mathcal{F}$ CS. Then A<sup>c</sup> is vague feebly open. By using theorem 1.1.10  $A^c \subseteq Vscl(Vint(A^c))$ . Taking complement on both sides  $(Vsint(Vcl(A)))^c \supseteq A^c$ .  $A^c \subseteq Vsint(Vcl(A^c))$ . Therefore  $A^c$  is a VFOS. By proposition 1.1.26. A is a VFCS. The following is an example of VFCS. **Example 1.1.28:** Let X = {a, b},  $\tau = \{0, 1, G\}$ , where G = {< x, [0.4, 0.7], [0.2, 0.4] >} then  $(X, \tau)$  be a VTS and let  $A = \{ < x, [0.3, 0.6], [0.6, 0.8] > \}$ . Here  $Vint(Vcl(Vint((A))) \subseteq A$ . Therefore,  $A = \{ < x, [0.3, 0.6], [0.6, 0.8] \}$  is a VFCS. Proposition 1.1.29: Every VCS is VFCS.

**Proof:** Let A be a VCS in X. Then A = Vcl(A).

Since Vint (A)  $\subseteq$  A, Vint(Vcl(A))  $\subseteq$  A  $\Rightarrow$  Vcl(Vint(Vcl(A)))  $\subseteq$  Vcl(A) = A.



www.ijprems.com

editor@ijprems.com

#### **INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT** AND SCIENCE (IJPREMS)

Vol. 04, Issue 02, February 2024, pp: 561-566

5.725

By proposition 1.1.25, A is a  $V\mathcal{F}CS$ .

The converse of the above theorem need not be true as shown in the example below **Example 1.1.30:** Let  $X = \{a, b\}, \tau = \{0, 1, G\}, t \in \{0, 1, G\},$ where  $G = \{\langle x, [0.4, 0.7], [0.2, 0.4] \rangle\}$  then  $(X, \tau)$  be a vague topological space and let  $A = \{ < x, [0.2, 0.6], [0.6, 0.7] > \}.$ Here A is a VFCS. **Proposition 1.1.31:** If A and B are any two VFCSs, then  $Vsint(Vcl(A)) \subseteq A$  and Vsint (Vcl(B))  $\subseteq$  B. **Proof:** By theorem 1.1.27, Vsint  $(Vcl(A)) \cap Vsint (Vcl(B)) \subseteq A \cap B$ . This implies  $Vsint(Vcl(A)) \cap Vcl(B)) \subseteq A \cap B$ . T his implies  $Vsint(Vcl(A \cap B)) \subseteq A \cap B$ . Hence  $A \cap B$  is a VFCS. Proposition 1.1.32: Finite intersection of VFCSs is a VFCS. **Proof:** Let  $\{A_i\}$  be a collection of VFCSs of a VTS  $(X, \tau)$ . Then by the definition 1.1.24 there exists a VCS V<sub>i</sub> such that  $V_i$ sint $(V_i) \subseteq A_i \subseteq V_i$  for each i. Now  $\cap$  V<sub>i</sub>sint(V<sub>i</sub>)  $\subseteq$   $\cap$  A<sub>i</sub>  $\subseteq$   $\cap$  V<sub>i</sub>  $\Rightarrow$  Vsint( $\cap$  V<sub>i</sub>)  $\subseteq$   $\cap$  A<sub>i</sub>  $\subseteq$   $\cap$  V<sub>i</sub> Hence  $\cap A_i$  is a VFCS. **Remark 1.1.32:** Union of any two VFCSs need not be a VFCSas shown in the example. **Example 1.1.33:** Let  $X = \{a, b\}, \tau = \{0, 1, G_1, G_2, G_3, G_4\}$  be a VT on X,

where  $G_1 = \{ \langle x, [0.5, 0.8], [0.5, 0.6] \rangle \}, G_2 = \{ \langle x, [0.4, 0.5], [0.6, 0.7] \rangle \}, \}$ 

 $G_3 = G_1 \cup G_2 = \{ < x, [0.5, 0.8], [0.4, 0.7] \}$ and

 $G_4 = G_1 \cap G_2 = \{ < x, [0.4, 0.5], [0.4, 0.6] \}$  and

let a VS A = {< x, [0.2, 0.5], [0.3, 0.6] >} and B = {< x, [0.5, 0.8], [0.6, 0.5] >} be two VFCSs in (X,  $\tau$ ) but A  $\cup$  B =  $\{ < x, [0.5, 0.8], [0.6, 0.6] > \}$  is not a VFCS in  $(X, \tau)$ .

## 2. CONCLUSION

This article has delved into the intricate concepts of vague feebly closed sets and vague feebly open sets within the framework of vague topological spaces. Through a rigorous exploration of these mathematical constructs, we have unveiled their properties and relationships and characteristics of vague topology. As we wrap up this study, it becomes evident that the investigation of vague feebly closed and open sets opens avenues for further research and exploration. We propose the following areas for future work: Extension to Higher Dimensions, Relation to Other Topological Concepts, Applications in Real-world Problems and Generalization to Different Vague Topological Spaces.

#### **3. REFERENCES**

- [1] Gau. W.L. and Buehrer. D.J, Vague sets, IEEE Transactions on Systems, Man, and Cybernetics, 23, 1993, 610-614.
- Ibraheem. D, On feebly generalized closed sets, Uruk Refereed Journal for Scientific Studies Issued by Al-[2] Muthana University (1), 2008.
- Ibraheem. D, On generalized feebly closed sets, Uruk Refereed Journal for Scientific Studies Issued by Al-[3] Muthana University (1), 2008, 131 – 139.
- [4] Maheshwari. S.N and Jain. P.G, Some new mappings, Mathematica, 24 (47), 1982, 53 - 55.
- [5] Vigneshwaran. M and Velmeenal. M, On RFG-Closed Sets in Topological Spaces, Journal of Informatics and Mathematical Sciences, 9(3), 2017, 799-814.