INTERNATIONAL JOURNAL OF PROGRESSIVE RESEARCH IN ENGINEERING MANAGEMENT AND SCIENCE (IJPREMS)

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editor@ijprems.com

Vol. 04, Issue 03, March 2024, pp: 791-797

e-ISSN : 2583-1062

Impact Factor: 5.725

ON THE LOMAX-DAGUM-X FAMILY OF DISTRIBUTION

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DOI: https://www.doi.org/10.58257/IJPREMS33020

ABSTRACT

In this research work, we construct a new class of asymmetric continuous probability distributions (Lomax-Dagum-X family), which serve as extensions of the Dagum-X family distribution. However, the density function and the cumulative distribution function of the constructed distribution (Lomax-Dagum-X family distribution) were obtained. We also, test the validity of the modified asymmetric probability distribution with corresponding plots of the distribution.

Keywords: Lomax-G, Dagum-X family

1. INTRODUCTION

Statistical distributions are commonly applied to describe real-world phenomena. Due to the usefulness of statistical distributions, their theory is widely studied and new distributions are developed. The interest in developing more flexible statistical distributions remains strong in the statistics profession. Many generalized classes of distributions have been developed and applied to describe various phenomena. A common feature of these generalized distributions is that they have more parameters. Johnson et al. (1994) stated that the use of four-parameter distributions should be sufficient for most practical purposes. According to these authors, at least three parameters are needed but they doubted any noticeable improvement arising from including a fifth or sixth parameter.

Statistical distributions find extensive application in describing real-world occurrences. Given their utility, the theory behind statistical distributions is extensively researched, leading to the development of novel distributions. The quality of statistical distribution is based on fitting the assumed probability distribution to the data. However, there are various issues where any of these distributions do not fit the data appropriately, especially in engineering, finance, medicine, and environmental hazards. Therefore, a significant effort has been made in developing different families of distributions Amani et al., (2023). The statistical profession continues to show a keen interest in creating even more adaptable distribution models. Statistical models play a crucial role in representing and analyzing datasets in practical applications. While traditional distributions, such as Weibull, Lomax, Uniform, gamma, log-normal, exponential, and beta, have been widely used, they may not always provide a satisfactory fit for complex datasets. Researchers have been actively developing new asymmetrical models offering greater adaptability and flexibility to address this limitation. These advancements often involve techniques, such as exponentiation, transformation, modification, T–X, beta, and gamma generator approaches, to generate more flexible asymmetrical distributions Sapkota et al., (2023).

The choice of appropriate distributions to be used on real-life data plays a fundamental role in improving the power, efficiency, and sensitivity of statistical tests. This is so because appropriate distributions lead to a good fit of the data. Therefore, good knowledge of the appropriate distribution to be used for a specific data set is essential. Probability distributions are very important in data analysis. They can be used to model a wide range of data shapes in applied fields. The Lomax (Lx) (also known as Pareto II) distribution has many applications in several areas such as income and wealth inequality, biological sciences, lifetime and reliability, engineering, and actuarial sciences. The Lx distribution has been applied in modeling real-life data in income and wealth, firm size, and reliability and life testing Atkinson and Harrison (1978); Corbellini et al., (2010); Harris (1968); Hassan and Al-Ghamdi (2009). Chahkandi and Ganjali (2009) showed that the Lx distribution belongs to the decreasing hazard rate (HR) family. More information about the Lx distribution can be explored in Anorld (1983); Johnson et al., (1994); Tahir et al., (2015). The procedure of adding new shape parameters for generalizing classical distributions is a well-known technique in the statistical literature. Hence, there are several extensions of the Lx distribution which are developed using well-known families to improve its flexibility and applicability in modeling different types of data. For example, Gupta et al. (1998) introduced the exponentiated Lomax, the Marshall–Olkin Lomax was proposed by Ghitany et al., (2007), the Kumaraswamy–Lomax by Lemonte and Cordeiro (2013), the Weibull–Lomax was studied by Tahir et al., (2015), the exponentiated half logistic-Lomax was introduced by Afify et al., (2017), the Fréchet Topp–Leone Lomax was proposed by Reyad et al., (2021), and the generalized linear failure rate Lomax by Afify et al., (2022). Recently, numerous classes of continuous probability/lifetime models have been proposed by different studies in the statistical literature. It has been proved that they are useful for adding performance, skewness, and flexibility of properties to the existing models. In other words, all of these approaches extend the classical baseline probability distributions by introducing additional parameter(s) to the baselines, thereby making the extended baselines much more flexible to fit a wide range of data from the practical situation. Some of these classical distributions include the Dagum-X family distribution by Amani et al., (2023) among others. However, real complex datasets characterized by asymmetric (kurtosis, skewness, N-shape, W-shapes etc) and bathtub in nature cannot be fitted/model with the aforementioned distributions as the estimate obtain from such distributions will be characterized by higher dispersion/variation. To overcome this challenge, some new distributions that are adaptive and flexible to accommodate datasets that are asymmetrical or bathtub in nature are required taking non-asymmetric distribution as baseline distribution. Therefore, in this research, some new distributions with the potential to accommodate asymmetrical or bathtub datasets will be proposed. Specifically, a new family distribution called Lomax-Dagum-G family of distribution will be proposed. Dagum (1977) proposed the distribution which is referred to as Dagum distribution which is based on the log logistic distribution by adding another parameter.

It is also called the generalized logistic-Burr distribution. There is both a three-parameter specification (Type I) and a four-parameter specification (Type II) of the Dagum distribution. Lukasiewicz et al. (2010) made a comparison among four models with various numbers of parameters: exponential, Weibull, Dagum, and Singh-Maddala to determine which model can represent the data that comes from the personal incomes in the USA by using some of the important measures such as the sum of squared residuals, the sum of absolute values of the residuals. Tahir et al., (2016) proposed a Weibull Dagum distribution where Its density function is very flexible and can be symmetrical, left-skewed, right-skewed, and reversed-J shaped. It has constant, increasing, decreasing, upside-down bathtub, bathtub, and reversed-J-shaped hazard rate.

Various structural properties are derived including explicit expressions for the quantile function, ordinary and incomplete moments, and probability-weighted moments. Tahir et al., (2016) proposed a Weibull Dagum distribution where Its density function is very flexible and can be symmetrical, left-skewed, right-skewed, and reversed-J shaped. It has constant, increasing, decreasing, upside-down bathtub, bathtub, and reversed-J-shaped hazard rate. Various structural properties are derived including explicit expressions for the quantile function, ordinary and incomplete moments, and probability-weighted moments. Rodrigues and Silva (2015) proposed Gamma Dagum distribution. application of the gamma-Dagum distribution to real data shows that the new distribution can be used quite effectively to provide better fits than the beta-Dagum, beta-Pareto, and Pareto confluent hypergeometric distributions. Oluyede and Rajasooriya (2013) proposed a new class of distributions called the Mc-Dagum distribution. An important motivation for the development Mc-Dagum distribution is the benefit of this class in its ability to fit skewed data that cannot properly be fitted in many other existing distributions.

2. METHOD FOR GENERATING A NEW FAMILY OF DISTRIBUTION

Let $r(t)$ be the pdf of a random variable $T \in [a,b]$, for $-\infty \le a \le b \le \infty$. Let $W(F(x))$ be a function of the cdf $F(x)$ of any random variable X so that $W(F(x))$ satisfies the following conditions.

1. $W(F(x)) \in [a,b]$
2. $W(F(x))$ is differentiable and monotonically non-decreasing
3. $W(F(x)) \to a$ as $x \to -\infty$ and $W(F(x)) \to b$ as $x \to \infty$.

Let X be the random variable with PDF $f(x)$ and CDF $F(x)$.

$$F(x) = \int_a^{W(F(x))} r(t)dt \qquad (1)$$

$$f(x) = \left[\frac{d}{dx} W(F(x))\right] r\left[W(F(x))\right] \qquad (2)$$

The definition of $W(F(x))$ depends on the support of the random variable T as follows:

1. When the support T is bounded: $W(F(x))$ can be defined as $F(x)$ or $F(x)^a$.

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2. when the support T is $[a,\infty]$, for $a \geq 0$: $W(F(x))$ can be defined as $-\log(1-F(x))$ or $F(x)/(1-F(x))$ or $-\log(1-F(x)^{a})$.
3. When the support of T is $(-\infty,\infty)$: $W(F(x))$ can be define as $\log\left[-\log(1-F(x))\right]$ or $\log\left[F(x)/(1-F(x))\right]$ by Alzaatreh et al., (2013).

2.1 Dagum-X Family of distribution

Amani et al., (2023) proposed a generalized Dagum distribution using the T-X method by Alzaatreh et al., (2013). The new family of Dagum distribution called Dagum-X, can be defined as follows: The CDF and PDF of the proposed New Dargum-X family of distribution by Amani et al. (2023) as in (3) and (4) respectively.

$$G(x;\theta,\lambda,\delta)=\left(1+\lambda\left(\frac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta} \tag{3}$$

The PDF is obtained by differentiating equation (3) concerning X as follows $g(x;\theta,\delta,\lambda)=\beta\theta\delta g(x)\dfrac{[G(X)]^{-\delta-1}}{\left[\bar{G}(X)\right]^{-\delta+1}}\left(1+\lambda\left(\dfrac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta-1}$ (4)

Where $x>0$ and $\delta,\theta>0$ are the shape parameters and $\lambda>0$ is the scale parameter respectively.

The survival function $S(x;\theta,\delta,\beta)$ and hazard rate function $h(x;\theta,\delta,\beta)$ of the Dagum-X family are obtained as in (5) and (6)respectively.

$$S(x;\theta,\lambda,\delta)=1-\left[1-\left(1+\lambda\left(\frac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta}\right] \tag{5}$$

$$h(x;\theta,\lambda,\delta)=\frac{\lambda\theta\delta g(x)\dfrac{[G(X)]^{-\delta-1}}{\left[\bar{G}(X)\right]^{-\delta+1}}\left(1+\lambda\left(\dfrac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta-1}}{1-\left[1-\left(1+\lambda\left(\dfrac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]} \tag{6}$$

Where $\lambda>0$, is the scale parameter and $\delta,\theta>0$ are the shape parameters respectively.

2.2 New Lomax -G Family

Sakpota et al., (2023) proposed a family of distribution called the New Lomax-G family of distribution which serves as the extension of Pareto type II (Lomax) distribution so that its support begins at zero by Lomax (1954).

The CDF and PDF of the proposed new family New Lomax-G Family distribution is given in (7) and (8) respectively.

$$G(x;\alpha,\beta)=\beta^{\alpha}\left[\beta-\log\left(F(x)\right)\right]^{-\alpha} \quad \text{and,} \tag{7}$$

$$g(x;\alpha,\beta)=\alpha\beta^{\alpha}f(x)F(x)^{-1}\left[\beta-\log\left(F(x)\right)\right]^{-(\alpha+1)} \tag{8}$$

Where $\alpha>0$ and $\beta>0$ are shape and scale parameters respectively.

Tthe survival function $S(x;\alpha,\beta)$ and hazard rate function $h(x;\alpha,\beta)$ of the lomax-x family is given in equation (9) and (10) respectively.

$$S(x;\alpha,\beta)=1-\beta^{\alpha}\left[\beta-\log\left(F(x)\right)\right]^{-\alpha} \tag{9}$$

$$h(x;\alpha,\beta)=\frac{\alpha\beta^{\alpha}f(x)F(x)^{-1}\left[\beta-\log\left(F(x)\right)\right]^{-(\alpha+1)}}{1-\beta^{\alpha}\left[\beta-\log\left(F(x)\right)\right]^{-\alpha}} \tag{10}$$

Where $x>0$ $\alpha>0$ and $\beta>0$ are the Shape and Scale parameters respectively.

2.3 The Proposed Lomax-Dagum-G Family Distribution

Then, the New Lomax-Dagum-G family has the cdf given by (11).

$$F(x;\alpha,\beta,\delta,\theta,\lambda)=\beta^{\alpha}\left[\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]^{-\alpha} \qquad x>0, \quad (11)$$

Where $\overline{G}(x)=1-G(x)$, $\delta>0$ and $\theta>0$ are the shape parameters, and $\lambda>0$ is the scale parameter.

To obtain the corresponding PDF, we differentiate (11) with respect to x and obtain (12)

$$\frac{d}{dx}F(x;\delta,\lambda,\theta)=f(x;\delta,\lambda,\theta)$$

$$y=\left[\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]^{-\alpha}$$ by keeping the β^{α} as a constant

$$f(x;\alpha,\beta,\theta,\delta,\lambda)=\alpha\theta\delta\lambda g(x)\frac{G(x)^{-\delta-1}}{\overline{G}(x)^{-(\delta+1)}}\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-1}\left[\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]^{-\alpha-1} \quad (12)$$

Where $\delta>0$, $\alpha>0$, $\beta>0$, and $\theta>0$ are the shape parameters, and $\lambda>0$ is the scale parameter.

4 Model Validity Check of the Proposed Lomax-Dagum-G family of distribution

To ensure the proposed probability distribution function (PDF) of the new family is valid and correct, it must satisfy the fact that;

$$\int_0^{\infty} f(x;\alpha,\beta,\theta,\delta,\lambda)=1 \quad (13)$$

The proof

$$\int_0^{\infty}\alpha\beta^{\alpha}\theta\delta\lambda g(x)\frac{G(x)^{-\delta-1}}{\overline{G}(x)^{-(\delta+1)}}\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-1}\left[\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}\right]^{-\alpha-1}dx=1 \quad (14)$$

Let

$$y=\beta-\log\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}, \quad k=\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-\theta}, \quad w=1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}, \quad t=\frac{G(x)}{\overline{G}(x)}$$

$$y=\beta-\log k,\ k=w^{-\theta}, \quad w=1+\lambda t^{-\delta} \ \text{ and } \ \frac{dt}{dx}=\frac{g(x)}{\overline{G}(x)^2}$$

By substituting the values of k,w and t we have

$$dx=-\frac{1}{\theta\delta\lambda g(x)G(x)^{-\delta-1}\overline{G}(x)^{-(\delta+1)}\left(1+\lambda\left(\frac{G(x)}{\overline{G}(x)}\right)^{-\delta}\right)^{-1}}dy$$

We then substitute for dx in (3.24) and given (3.25) respectively.

$$=-\int_0^{\infty}\alpha\beta^{\alpha}\left[\beta-\log k\right]^{-\alpha-1}dk \quad (15)$$

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Recall that $k = \left(1+\lambda\left(\dfrac{G(x)}{\bar{G}(x)}\right)^{-\delta}\right)^{-\theta}$

Then we simplify the (3.25) at when $x \to \infty$, and when $x \to 0$

$$\int_0^\infty f(x;\alpha,\beta,\theta,\delta,\lambda) = -\alpha\beta^\alpha \frac{\left[\beta^{-\alpha} - \log k^{-\alpha}\right]_0^\infty}{-\alpha} \qquad (16)$$

$$\int_0^\infty f(x;\alpha,\beta,\theta,\delta,\lambda) = (1-0)-(1-1) = 1 \qquad (17)$$

Hence, the model in equation (12) is a valid probability density function.

2.4 Plots CDF and PDF of Lomax-Dagum X family

Plots of cumulative distribution function and probability density function of Lomax-Dagum X family for selected/varying values for the five parameter to study its behaviour are given in Figures 1 and 2 and furthermore, for a similarity, $a = \alpha, b = \beta, c = \delta, d = \theta \; and \; e = \lambda$.

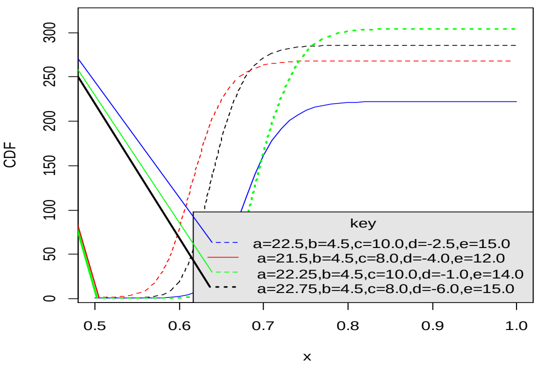

Figure 1.

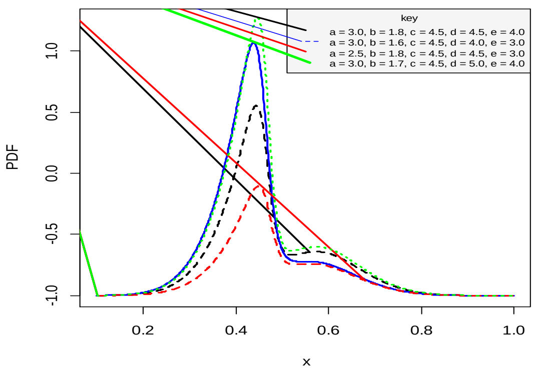

Figure 2.

Figure (1) displays the distribution function of LD-X family for different values of the shape and scale parameters. Figure (2) displays the density function of LD-X family for different values of the shape and scale parameters. It is both left and right skewed distribution and has different level kurtosis which shows the flexibility of the distribution for modeling asymmetric datasets

3. CONCLUSION

In this paper, we construct a new classes of asymmetric continuous probability distributions (Lomax-Dagum-X family) which serve as extension of Dagum-X family. However, density function and the cumulative distribution function of the constructed distribution (Lomax-Dagum-X family were obtained. We also, proceed to test the validity of the new asymmetric distribution. Explicit expressions for some basic statistical properties of these distributions such as Moments, Moment Generating Function, Quantile function and order statistics was derived.

4. REFERENCES

[1] Afify, A. Z., Yousof, H. M., Cordeiro, G. M., Ortega, E. M. M., & Nofal, Z. M. (2016): The Weibull Fréchet distribution and its applications. Journal of Applied Statistics, 43 (14), 2608–2626.

[2] Afify, A.Z.; Marzouk, W.; Al-Mofleh, H.; Ahmed, A.H.N.; Abdel-Fatah, N.A. (2022). The extended failure rate family: Properties and applications in the engineering and insurance fields. Pakistan Journal of Statistics, 38, 165–196.

[3] Afify, A.Z.; Yousof, H.M.; Nadarajah, S. (2017). The beta transmuted-H fam ily for lifetime data. Stat. Its Interface 2017, 10, 505–520.

[4] Ahmad, Z. (2018):, The Zubair-G Family of Distributions: Properties and Applications, Annual Data. Science. Vol. 7 195-208. https://doi.org/10.1007/s40745-018-0169-9.

[5] Ahsan-ul-Haq, M., Ahmed, J., Albassam, M. and Aslam, M. (2022): Power Inverted Nadarajah–Haghighi Distribution: Properties, Estimation, and Applications Hindawi Journal of Mathematics. Vol. 22, 1-10.

[6] Al-Aqtash, R., Famoye, F. and Lee, C. (2015). On generating a new family of distributions using the logit function: Journal of Probability and Statistical Science: 13 (1):135-152

[7] Aldahlan, M. A., Afify, A, Z. and Ahmed, A. N. (2019). The odd inverse Pareto-G class: properties and applications. Journal of Nonlinear Sciences and Applications; 12: 278-290.

[8] Alzaatreh, A., Lee, C., and Famoye, F., (2013): A New Method for Generating Families of Continuous Distributions, Metron. 71, 63-79. https://doi.org/10.1007/s40300-013-0007-y.

[9] Amani S.A., Huda, A., and Aisha, F., (2023): The New Dagum-X family of distribution properties and application. International Journal of Analysis and Applications.21:45.

[10] Arnold, B.C. (1983). Pareto Distributions; International Cooperative Publishing House: Fairland, MD, USA, 1983.

[11] Atkinson, M.; Harrison, A.J. (1978). Personal Wealth in Britain; CUP Archive: Cambridge, UK.

[12] Basheer, A.M. (2019) Alpha Power Inverse Weibull Distribution with Reliability Applications. Journal of Taibah University for Science, 13, 423-432. https://doi.org/10.1080/16583655.2019.1588488

[13] Chahkandi, M.; Ganjali, M. (2009). On some lifetime distributions with decreasing failure rates. Comput. Stat. Data Anal. 53, 4433–4440.

[14] Chrisogonus K.O., George A. Osuji, and Samuel U. Enogwe (2020), “Inverted Power Rama Distribution with Application to Life Time Data”, AJPAS, 9(4): 1-21, Article no.AJPAS.62214

[15] Chrisogonus K.O., George A.O, Samuel U.E., Michael, C.O., and Victor, N.I., (2021): Exponentiated Rama distribution properties and application. Journal of mathematical theory and modeling. Vol. 11 No.1. 2224-5804.

[16] Corbellini, A.; Crosato, L.; Ganugi, P.; Mazzoli, M. (2010). Fitting Pareto II distributions on firm size: Statistical methodology and economic puzzles. In Advances in Data Analysis: Theory and Applications to Reliability and Inference, Data Mining, Bioinformatics, Lifetime Data, and Neural Networks; Springer: Berlin/Heidelberg, Germany, pp. 321–328.

[17] Cordeiro, G. M., Cancho, V. G., Ortega, E. M., and Barriga, G. D. (2016). A model with long-term survivors: negative binomial Birnbaum-Saunders.

[18] Dagum, C. (1977). ‘A New Model of Personal Income Distribution: Specification and Estimation.’ Economie Applique’e, Tomo XXX, No. 3, pp. 413–436.

[19] Deniz E.G., and Ojeda E.C., (2011). The discrete Lindley Distribution-Properties and Applications. Journal of Statistical Computing and Simulations 81(11):1405-1416.

[20] Dey, S., Ghosh, I. and Kumar, D. (2018) Alpha-Power Transformed Lindley Distribution: Properties and Associated Inference with Application to Earthquake Data. Annals of Data Science, 6, 623-650. https://doi.org/10.1007/s40745-018-0163-2

[21] Domma, F., (2004): Kurtosis Diagram for the Log-Dagum Distribution, Stat. Appl. 2, 3–23.

[22] Domma, F., and Perri, P.F., (2008): Some Developments on the Log-Dagum Distribution, Journal of Statistical Methods Application. 18,205-220. https://doi.org/10.1007/s10260-007-0091-3.

[23] Korkmaz, M. C., Alizadeh, M., Yousof, H.M. and Butt, N. S. (2018b). The generalized odd Weibull generated a family of distributions: statistical properties and applications. Pakistan Journal of Statistical Operation Research vol.XIV. No. 3:541-556.

[24] Korkmaz, M. C., Yousof, H. M., Hamedani, G. G. and Ali, M. M. (2018a). The Marshall-Olkin generalized G Poison family of distributions. Pakistan Journal of Statistics; 34(3):251-267.

[25] Kotz, S., and Dorp, J. R. (2004): Beyond beta: Other continuous families of distributions with bounded Support and applications (pp. 289). Singapore: World Scientific.

[26] Kotz, S., and Nadarajah, S.,(2000): Extreme value distributions: theory and applications. World Scientific.

[27] Krishna, A.; Maya, R.; Chesneau, C.; Irshad, M.R. The Unit Teissier Distribution and Its Applications. Mathematical and Computational. Application 2022, 27, 12. https://doi.org/10.3390/mca27010012

[28] Lemonte, A.J.; Cordeiro, G.M. (2013). An extended Lomax distribution. Statistics, 47, 800–816.

[29] Oguntunde, P. E., Khaleel, M. A., Ahmed, M. T., Adejumo, A. O., and Odetunmibi, O. A. (2017). A new generalization of the Lomax distribution with increasing, decreasing, and constant failure rate. Modeling and Simulation in Engineering, 2017, Article ID 60431696. doi:10.1155/2017/6043169.

[30] Oguntunde, P.E., Khaleel, M.A., Ahmed M.T. and Okagbue, H. I., (2019): The Gompertz Fréchet distribution, Properties and applications. Cogent Mathematics & Statistics, 6:1, 1568662, https://doi.org/10.1080/25742558.2019.1568662

[31] Okereke, E.W., and Uwaeme, O.R., (2018): Exponentiated Akash and its applications. Journal of the Nigerian Statistical Association. Vol. (30) pages 1-13.

[32] Oluyede, B. O., Huang, S., and Pararai, M. (2014). 'A New Class of Generalized Dagum Distribution with Applications to Income and Lifetime Data.' Journal of Statistical and Econometric Methods, Vol. 3, No. 2, pp.125-151.