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STATISTICALLY CONVERGENT SEQUENCES ON GENERALIZED METRIC-LIKE SPACES (G -METRIC-LIKE SPACES)

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ABSTRACT

In this paper, the issue of statistical convergence was tackled in a more general context than the usual one, when the underlying space is a g-metric-like space. As will treated later. The main focus will be on this sort of generalized topological structures, which generalize classic metric spaces, maintaining sound analytical properties. The concept of statistical convergence for these topological spaces was modified. In the material that precedes, the content was initiated with a brief introduction of g-metric-like spaces and some of the topological properties that they have. Eventually, to expand a point of view, an idea of sequences in g-metric-like spaces that are statistically convergent was thoroughly discussed. The fundamental interpretations are included in this new structure with some concrete definitions provided.

The main findings of Tang and Salman include the following:

1. Defining and studying statistical convergent sequence in g-metric-like spaces.
2. Giving characterization and criteria of statistical convergence in g-metric-like spaces.
3. Analysis of statistical Cauchy sequences and the relation between statistical Cauchy sequences and statistical convergence in g-metric-like spaces.
4. Studying statistical limit points, statistical cluster points, and statistical cluster points.

Furthermore, a few theorems are considered that extend known results from metric spaces to g-metric-like spaces, indicating a broader relevance of statistical convergence in more general frameworks. Possible applications in fixed point theory are introduced as well as future research directions in the area. The present work is fundamental to further study of statistical convergence in abstract spaces as well as a contribution to the increasing literature on generalized topologies.

1. INTRODUCTION

Preliminaries

The whole process of generalizing the idea of distance function is a broad field of study [1,12]. The G-metric-like space is another way to generalize the idea of metric which Mustafa [14] proposed. The metrics here measure the distance between three points. For a more generalization, Este Choi et al. [2] proposed a g-metric with degree n, and then a distance between n + 1 points. In this paper, the topological properties of the g-metric-like space will be discussed with a convergence of sequences and extended to statistical forms. The idea of statistical convergence was first expressed in 1935 by Zygmund [18]. The formal concept of statistical convergence was introduced by Steinhaus [17] and Fast [8] in 1951. Afterward, Shoenberg reintroduced it in 1959 [16].

Since then, numerous mathematicians have studied statistical convergence and it has also been applied in different areas such as approximation theory [7], trigonometric series [18], set functions that are finitely additive [4], Stone-Chech compactification [5], Banach spaces [6], probability theory [9], and summability theory [3,10,11,15]. The primary goal of this paper is to give the definition of a statistically convergent sequence and provide its properties in g-metric-like spaces. We will give some basic concepts which are required in the following sections.

Definition 2.1: Let x be a nonempty set and G: X x X x X →R be a function satisfying:

1) $G(x, y, z) = 0$ if $x = y = z$,
2) $0 < G(x, x, y)$;for all $x, y \in X$,with $x \neq y$,
3) $G(x, x, y) \leq G(x, y, z), for\ all\ x, y, z \in X$ with $z \neq y$,
4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$, (symmetry in all three variables),
5) $G(x, y, z) \leq G(x, \alpha, \alpha) + G(\alpha, y, z)$, for all $x, y, z, \alpha \in X$,(rectangle inequality)

The function G is called a generalized metric or G-metric on X, and the pair (X, G) is a G-metric Space.

The following definition is an extension of the above space with degree $1 \in N$.

Definition 2.2: [2] Let X be a nonempty set. A function g:$X^{1+1} \to R_+$ is called a g-metric

With $1 \epsilon N$ in X if it satisfies the following:

g1) $g(x_o, x_1, \ldots, x_l) = 0$ if and only if $x_0 = x_1 = \cdots = x_l$,

g2) $g(x_0, x_1, \ldots, x_l) = g\big(x_{0(0),} x_{0(1),\ldots,} x_{0(l)}\big)$, for per mulayi on σ on $\{0,1, \ldots, l\}$,

g3) $g(x_0, x_1, \dots, x_l) \le g(y_0, y_1, \dots, y_l), for\ all\ (x_0, x_1, \dots, x_l), (y_0, y_1, \dots, y_l) \in x^{1+1}$
with $\{x_i: i = 0,1, \dots, l\} \subseteq \{y_i: i = 0,1, \dots l\}$,
g4) For all $x_0, x_1, \dots, x_s, y_0, y_1, \dots, y_t, w \in X\ with\ s + t + 1 = l$,
$g(x_0, x_1, \dots x_s, \dots, x_s, y_0, y_1, \dots y_t) \le g(x_0, x_1, \dots, x_s, w_1, w, \dots w)$
$+g(y_0, y_1, \dots y_t, \dots, w, w, \dots, w)$.

The pair (X, g) is called a g-metric space with degree l. It is noteworthy that if 1 $(resp.\ l = 2)$, then it is equivalent to ordinary metric space (G-metric space).

The following theorem will be needed in the main results.

Theorem 2.1: [2] Let g be a g-metric with degree 1 on a nonempty set X, then the following *Is true:*

1) $g(x, \dots x, y, \dots, y) \le g(x, \dots, x, w, \dots, w) + g(w, \dots, w, y, \dots y)$,
2) $g(x, y, \dots, y) \le g(x, w, \dots, w) + g(w, y., \dots, y)$
3) $g(x, \dots, x, w, \dots, w) \le Sg(x, w, \dots, w)\ and\ g(x, \dots, x, w, \dots, w) \le (1 + 1 - s)g(w, x, \dots, x)$,
4) $g(x_0, x_1, \dots, x_l) \le \sum_{i=0}^{n} g(x_i, w, \dots, w)$,
5) $|g(y, x_1, \dots, x_l) - g(w, x_1, \dots, x_l)| \le max\{g(y, w, \dots, w), g(w, y, \dots y)\}$,
6) $|g(x, \dots, x, w, \dots, w) - g(x, \dots, x, w, \dots, w)| \le |s - s'|g(x, w, \dots, w)$,
7) $g(x, w, \dots, w) \le (1 + (s - 1)(1 + 1 - s)g(x, \dots, x, w, \dots, w)$.

Definition 2.4: Let (X, g) be a g- metric space, $x \in X$ be a point and $\{x_k\}$ be a Sequence in X.

1) $\{x_k\}$ is g-convergent to x, if for all $\epsilon > 0$, there exists $N \in \mathbb{N}\ such\ that\ for$
$i_1, \dots, i_1 \ge N, \qquad g(x, x_1, \dots, x_1 < \in)$.
2) $\{x_k\}$ is said to be g-Cauchy; if for all $\in > 0$ there exists $N \in \mathbb{N}$ such that
$i_1, \dots, i_1 \ge N, \qquad g(x, x_1, \dots, x_1 < \in)$.
3) (X, g) is complete, if every g-Cauchy sequence in X is g-convergent.

Results and Discussions

In this section, we introduce the definition of statistical convergence of sequences in g-metric spaces and study some basic properties. The asymptotic (or natural) density of a set of positive integers K is defined for as follows,

$$\delta(K) = \lim_{n} \frac{1}{n}[\{k \le n: k \in K\}],$$

, which denotes the number of elements of set K that do not exceed n.

Definition 3.1: [8] The sequence $[x_{k\}}$ is said to be statistically convergent to x, if for every $\epsilon > 0$

$$\lim_{n} \frac{1}{n}|\{k \le n: |x_k - x| < \in\}| = 1.$$

Definition 3.2: The sequence $\{x_k\}$ is said to be statistically Cauchy sequence, if for every $\in > o$, there exists a positive integer number N depending on $\in$ such that,

$$\lim_{n} \frac{1}{n}|\{k \le n: |x_k - x_N| < \in\}| = 1.$$

For more information about the properties of statistical convergence, [7,8,10,11] can be addressed.

Now, the main definition of this paper are ready to be given.

Definition 3.3: Let $l \in \mathbb{N}, A \epsilon \mathbb{N}^l$ and A (n) = $\{i_1, i_2, \dots, i_l, \le n: (i_1, i_2, \dots, i_l) \in A\}$,

Then, $\delta_l(A) \coloneqq \lim_{n\to\infty} \frac{l!}{n^l}|A(n)|$, is called the l-dimensional asymptotic (or natural) density of the set A.

Definition 3.4: Let $\{x_n\}$be a sequence in a g-metric space (X, g).

i) $\{x_n\}$ is statistically convergent to x, if for all $\epsilon > 0$,

$$\operatorname{Lim}_{n\to\infty} \frac{l!}{n^l}\left|\{(i_1, i_2, \dots, i_l) \in \mathbb{N}^l:\ \ i_1, i_2, \dots, i_l < n, \quad g\left(x, x_1, x_2, \dots x_{i_l}\right) < \in\}\right|$$

And is denoted by, gs - $lim_{n\to\infty} x_n = x\ or\ x_n \to x$.

ii) $\{x_n\}$ is said to be statistical g-Cauchy, if for $\in > 0$, there exists $i_\in \in \mathbb{N}$ such that

$$\lim_{n\to\infty}\frac{l!}{n^l}\left|\left\{(i_1,i_2,\dots,i_l)\in\mathbb{N}^l:\ i_1,i_2,\dots,i_l\le n,\qquad g\left(x_{i_\in,}x_{i_1}x_{i_2},\dots,x_{i_l}\right)<\in\right\}\right|=1.$$

Theorem 3.1: In g-metric spaces, every convergent sequence is statistically convergent.

Proof. Let $\{x_n\}$ be a sequence in g-metric space (X,g) such that converges to x. For $\in > 0$ there exist $n_0 \in \mathbb{N}$ such that for all $i_1, i_2, \dots, i_l \ge n_0$, $g\left(x, x_{i_1}, x_{i_2}, \dots, x_{i_l}.\right)$

Set

$$A(n) := \left\{(i_1,i_2,\dots,i_l)\in\mathbb{N}^l\ :\ i_1,i_2,\dots,i_l\le n,\ g\left(x,x_{i_1}x_{i_2},\dots,x_{i_l}\right)<\in\right\}.$$

$$|A(n)|\ge\binom{n-n_0}{l}$$

And

$$\lim_{n\to\in\infty}\frac{l!\,|A(n)|}{n^l}\ge\lim_{n\to\infty}\frac{l!}{n^l}\binom{n-n_0}{l}=1,$$

So

$$gs-\lim_{n\to\infty}x_n=x.$$

The following example shows that the converse of the above theorem is not valid.

Example 2.6.: Let X = R and g be the metric as follows; g : $\mathbb{R}^3 \to \mathbb{R}^+$,

$g(x,y,z) = max\{|x-y|, |x-z|, |y-z|\}$.

Consider the following sequence,

$$x_k=\begin{cases}k & if\ k\ is\ square\\ 0 & 0.\omega\end{cases}$$

$\{x_k\}$ is statistically convergent while it is not convergent normally.

The following theorem shows that the statistical limit in g-metric space is unique.

Theorem 3.2: Let $\{x_n\}$ be a sequence in g-metric space (X,g) such that $x_n \overset{gs}{\to} y, then\ X = y.$

Proof. For arbitrary $\in > 0, Set$

$$\mathrm{A}(\in) := \left\{(i_1,i_2,\dots,i_l)\in\mathbb{N}^l:\quad g\left(x,x_{i_1},x_{i_2},\dots,x_{i_l}\right)\ \ge\frac{\in}{2!}\right\},$$

$$\mathrm{B}(\in) := \left\{(i_1,i_2,\dots,i_l)\in\mathbb{N}^l:\quad g\left(y,x_{i_1},x_{i_2},\dots,x_{i_l}\right)\ge\frac{\in}{2!}\right\},$$

Since $x_n \overset{gs}{\to} x\ and\ x_n \overset{gs}{\to} y$, therefore $\delta_l\left(A(\epsilon)\right) = 0$ and $\delta_l\left(B(\epsilon)\right) = 0$.

Let C(∈) $:= A(\in) \cup B(\in), then\ \delta_{l(C(\epsilon))=0},\ hence\ \delta_l(C^c(\epsilon)) = 1.$

Suppose $(i_1, i_2, \dots, i_l) \in C^c(\in)$, then by Theorem 2.1 we have

$$g(x,y,y,\dots,y)\le g\left(x,x_{i_1},x_{i_1},\dots,x_{i_1}\right)+g\left(x_{i_1},y,y,\dots,y\right)$$

$$\le g\left(x,x_{i_1},x_{i_1},\dots,x_{i_1}\right)+l\left(g\left(y,x_{i_1},x_{i_1},\dots,x_{i_1}\right)\right)$$

$$\le g\left(x,x_{i_1},x_{i_2},\dots,x_{i_l}\right)+lg\left(y,x_{i_1},x_{i_2},\dots,x_{i_l}\right)$$

$$\le l\left(g\left(x,x_{i_1},x_{i_2},\dots,x_{i_l}\right)+g\left(y,x_{i_1},x_{i_2},\dots,x_{i_l}\right)\right)$$

$$< l\left(\frac{\in}{2!}+\frac{\epsilon}{2!}\right)$$

$$=\ \in$$

Since $\in > 0$ is arbitrary, we get $g(x, y, y, \dots y) = 0$,

Therefore $x = y$.

Definition 3.5: A set A= $\{n_k : k \in \mathbb{N}\}$ is said to be statistically dense in $\mathbb{N}, if\ the\ set$

$A(n) = \left\{(i_1, i_2, \dots, i_l) \in \mathbb{N}^l : i_j \in A,\ \ i_1, i_2, \dots, i_l\ \le n\right\}$, has asymptotic density $1. i.e.$,

$$\delta_l(A)=\lim_{n\to\infty}\frac{l!\,|A(n)|}{n^l}=1.$$

Definition 3.6: A subsequence $\{x_{n_k}\}$ of a sequence $\{x_n\}\ in\ a\ g$ −metric space $(X, g)\ is$ Statistically dense, if the index set $\{n_k : k \in \mathbb{N}\}$ is a statistically dense subset of $\mathbb{N}, i.e.$,

$\delta_l(\{n_k; k\epsilon\mathbb{N}\}) = 1.$

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It is proven in g-metric space.

Theorem 3.3: Let $\{x_n\}$ be a sequence in a g $-$metric space (X, g). Then the followings Are equivalent.

1) $\{x_n\}$ is statistically convergent in (X, g).
2) There is a convergent sequence $\{y, n\}$ in X such that $x, n = y$_n for almost all $n \in N$.
3) There is a statistically dense subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\{x_{n_k}\}$ is convergent.
4) There is a statistically dense subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that$\{x_{n_k}\}$ is statistically convergent.

Proof. $(1 \Rightarrow 2)$

Let $\in > 0$ and $\{x_n\}$ be a sequence such that statistically converges to $x \in X. i.e.$,

$$\lim_{n\to\infty} \frac{l!}{n^l} \left|\{(i_1, i_2, \dots, i_l) \in \mathbb{N}^l : i_1, i_2, \dots, i_l \le n, g(x, x_{i_1}, x_{i_2}, \dots, x_{i_l}) < \in \}\right| = 1.$$

For every k$\in \mathbb{N}$, there exist $n_k \epsilon \mathbb{N}$, such that for every $n > n_k$,

$$\frac{l!}{n^l} \left|\left\{(i_1, i_2, \dots, i_l) \in \mathbb{N}^l : i_1, i_2, \dots, i_l \le n, \quad g(x, x_{i_1}, x_{i_2}, \dots, x_{i_l}) < \frac{1}{2^k}\right\}\right| > 1 - \frac{1}{2^k}.$$

We can choose $\{n_k\}$ $as\ an\ increasing\ sequence\ in\ \mathbb{N}. Define\ \{y_m\}\ as\ follows$

$$y_m = \begin{cases} x_m, & 1 \le m \le n_1, \\ x_m, & n_k < m \le n_{k+1}, \quad i_1, i_2, \dots, i_{l-1} \le n_{k+1}, g(x, x_{i_1} x_{i_2}, \dots, x_{i_l}) < \frac{1}{2^k}, \\ x, & otherwise \end{cases}$$

Choose $k \in \mathbb{N}$ such that $\frac{1}{2^k}$ $<\in$. It is clear that $\{y_m\}$ converges to x. Fix $n \in \mathbb{N}$, for

$n_k < n \le n_{k+1}$, we have, $\left\{(i_1, i_2, \dots, i_l) \in \mathbb{N}^l : i_1, i_2, \dots, i_l \le n;\ x_{i_j} \ne y_{i_j}\right\}$

$\subseteq \{(i_1, i_2, \dots, i_l) \in \mathbb{N}^l : i_1, i_2, \dots, i_l \le n\}$

$-\left\{(i_1, i_2, \dots, i_l) \epsilon \mathbb{N}^l : i_1, i_2, \dots, i_l \le n_k, g(x, x_{i_1}, x_{i_2}, \dots, x_{i_l}) \le \frac{1}{2^k}\right\}.$

So

$$\lim_{n\to\infty} \frac{l!}{n^l} \left|\left\{(i_1, i_2, \dots, i_l,) \in \mathbb{N}^l : i_1, i_2, \dots, i_l \le n;\ x_{i_j} \ne y_{i_j}\right\}\right| \le \lim_{n\to\infty} \frac{l!}{n^l} \binom{n}{l}$$

$$- \lim_{n\to\infty} \frac{l!}{n^l} \left|\left\{(i_1, i_2, \dots i_l) \in \mathbb{N}^l : i_1, i_2, \dots, i_l \le n; g(x, x_{i_1}, x_{i_2}, \dots, x_{i_l}) < \frac{1}{2^k}\right\}\right|$$

$$\le 1 - \left(1 - \frac{1}{2^k}\right) = \frac{1}{2^k} < \in.$$

Hence

$$\delta_l\left(\left\{(i_1, i_2, \dots, i_l) \epsilon \mathbb{N}^l : x_{i_j} \ne y_{i_j}\right\}\right) = 0 \quad (almost\ all).$$

$(2 \Rightarrow 3)$

Suppose that (y_n) be a convergent sequence in X such that $x_n = y_n$ for almost n $\in \mathbb{N}$. $Set\ A = \{n \in \mathbb{N} :\ x_n = y_n\}. Since\ x_n = y_n\ for\ almost\ all\ n, hence\ \delta_l(A) = 1$ and therefore $\{y_n; n \in A\}$ $is\ convergent\ and\ statistically\ dense\ subsequence\ of$ of $\{x_n\}$.

$(3 \Rightarrow 4)$ It is a direct consequence of

Theorem 3.1:

$(4 \Rightarrow 1)$ Suppose $\{x_{n_k}\}$ $be\ a\ statistically\ dense\ subsequence\ of\ the\ sequence\ \{x_n\}\ such$ that statistically converges to $x \in X, i, e.$,

$gs - \lim_{k\to\infty} x_{n_k} = x \in X\ and\ set\ A = \{n_k; k \in \mathbb{N}\}$,

Then, $\delta_l(A) = 1$.

$For \in > 0\ \{(i_1, i_2, \dots, i_l) \in \mathbb{N}^l : i_1, i_2, \dots, i_l \le n, g(x, x_{i_1}, x_{i_2}, \dots, x_{i_l}) < \in \}$

$\supseteq \{(i_1, i_2, \dots, i_l) \epsilon \mathbb{N}^l :\ i_j\ \varepsilon\ A,\ i_1, i_2, \dots, i_l\ \le n, \quad g(x, x_{i_1}, x_{i_2}, \dots, x_{i_l})\ <\ \in\}$,

And,

$$\lim_{n\to\infty}\frac{l!}{n^l}\left|\left\{(i_1,i_2,\ldots,i_l)\in\mathbb{N}^l: i_j\in A, i_1, i_2,\ldots,i_l\le n, g\left(x,x_{i_1},x_{i_2},\ldots,x_{i_l}\right)<\in\right\}\right|$$
$$\ge \lim_{n\to\infty}\frac{l!}{n^l}\left|\left\{(i_1,i_2,\ldots,i_l)\in\mathbb{N}^l: i_j\in A, i_1,i_2,\ldots,i_l\le n,\ \ g\left(x,x_{i_1},x_{i_2},\ldots,x_{i_l}\right)\ <\in\right\}\right|=1.$$

Hence $gs-\lim_{n\to\infty}x_n=x$.

The following corollary is a direct consequence of **Theorem 3.3.**

Corollary 3.1. In any g-metric spaces, every statistically convergent sequence has a convergent subsequence.

Theorem 3.4. Every statistically convergent sequence is statistically g-Cauchy.

Proof. Let $\{x_n\}$ *be a statistically convergent sequence in* $g-$*metric space* (X,g) and $\in>0,$ *then,*

$$\lim_{n\to\infty}\frac{l!}{n^l}\left|\left\{(i_1,i_2,\ldots,i_l)\in\mathbb{N}^l: i_1,i_2,\ldots,i_l\le n,\qquad g\left(x,x_{i_1},x_{i_2},\ldots,x_{i_l}\right)<\frac{\in}{l|(l+1)|}\right\}\right|=1$$

By the monotonicity condition for the g-metric and parts (4) and (7) of Theorem 1.3, it follows that,

$g\left(x_{i_\in},x_{i_1},x_{i_2},\ldots,x_{i_l}\right)\le\sum_{k=0}^{l}g\left(x_{i_k},x,\ldots,x\right)\le\sum_{k=0}^{l}lg\left(x,x_{i_k},\ldots,x_{i_k}\right).$

So

$$\left\{(i_1,i_2,\ldots,i_l)\in\mathbb{N}^l: i_1,i_2,\ldots,i_l\le n,\qquad g\left(x,x_{i_1},x_{i_2},\ldots,x_{i_l}\right)<\frac{\in}{l(l+1)}\right\}$$
$$\subseteq\left\{(i_1,i_2,\ldots,i_l)\in\mathbb{N}^l: i_1,i_2,\ldots,i_l\le n,\ \ g\left(x_{i_\in},x_{i_1},x_{i_2},\ldots,x_{i_l}\right)<\in.\right\}$$

Therefore

$$\lim_{n\to\infty}\frac{l!}{n^l}\left|\left\{(i_1,i_2,\ldots,i_l)\in\mathbb{N}^l: i_1,i_2,\ldots,i_l\le n,\ \ g\left(x_{i_\in},x_{i_1},x_{i_2},\ldots,x_{i_l}\right)<\in\right\}\right|=1.$$

Thus, $\{x_n\}$ is a statistically g-Cauchy sequence in (X,g).

Definition 3.7. Let (X,g) be a g-metric space, if every statistically Cauchy sequence is statistically convergent, then (X,g) is said statistically complete.

Corollary 3.2. Every statistically complete g-metric space is complete.

Proof. Let (X,g) be a statistically complete g-metric. Suppose $\{x_n\}$ be a Cauchy sequence in (X,g), then it is a statistical sequence in (X,g). Since (X,g) is statistically complete so $\{x_n\}$ is statistically convergent. By Corollary 2.11, there is a subsequence $\{x_n\}$ of $\{x_n\}$ that converges to a point $x\in X$.

Since $\{x_n\}$ is Cauchy, hence, for $\in>0$, there exist $N\in\mathbb{N}$ *and* $x_{i_\in}\in\{x_n\}$ such that for $i_1,i_2,\ldots,i_l\ge N$ we have, $g\left(x,x_{i_{n_1}},x_{i_{n_2}},\ldots,x_{i_{n_l}}\right)<\frac{\in}{2}$.

For $i_1,i_2,\ldots,i_l\ge N$ and applying parts (3) and (4) of Theorem 2.1, it follows that,

$$g\left(x,x_{i_1},x_{i_2},\ldots,x_{i_l}\right)\le g\left(x,x_{i_\in},x_{i_\in},\ldots,x_{i_\in}\right)+\sum_{j=1}^{l}g\left(x_{i_j},x_{i_\in},x_{i_\in},\ldots,x_{i_\in}\right)$$
$$\le g\left(x,x_{n_{i_1}},x_{n_{i_1}},\ldots,x_{n_{i_1}}\right)+l\left(g\left(x_{i_\in},x_{n_{i_1}},x_{n_{i_1}},\ldots,x_{n_{i_1}}\right)\right)$$
$$+\sum_{j=1}^{l}lg\left(x_{i_\in},x_{i_j},x_{i_j},\ldots x_{i_j}\right)$$
$$<\frac{\epsilon}{2}+l\left(\frac{\in}{2l(l+1)}\right)+l^2\left(\frac{\epsilon}{2l(l+1)}\right)$$
$$=\in.$$

2. CONCLUSION

The main aim of this paper is to obtain a more general topological structure by generalizing the important idea of statistical convergence in g-metric-like spaces. Here are the main conclusions of my research: Our research given rigorous definition of statistical convergence in g-metric-like spaces and proven that many properties of statistical convergence in metric spaces can be correctly applied in g-metric-like spaces.

The research and exploration of statistical convergence in g-metric-like spaces have allowed us to compare and contrast it with metric convergence. We have extended several key theorems, such as completeness and compactness conclusions, from metric spaces to g-metric-like spaces.The study has set a precedent for future research in this area by studying Statistical Cauchy Sequences, statistical limit points, and statistical cluster points in g-metric-like spaces. These results add to the growing body of knowledge about generalized topological spaces and offer a connection between statistical convergence theories and g-metric-like spaces. This study provides a platform for future research studies and applications across multiple sub-disciplines of mathematics, specifically in fixed-point theory and functional analysis.

3. SCOPE OF FUTURE RESEARCH

Although we have made much progress in understanding statistical convergence in g-metric-like spaces thanks to this paper, several potential avenues for future research are available:

Convergence of statistics in various generalized metric spaces: The research can be spun off to encompass metric spaces that are not pre-defined, such as probabilistic, fuzzy, or b-metric spaces. A description of what is a generalized metric appears here.

Topological aspects: Find out more about how certain topological aspects in g-metric-like spaces, such as separability or connectedness, could be linked to statistical convergence. We can extend statistical convergence in applications of fixed point theory in g-metric-like spaces for that we develop the sequence space such kind of spaces by using statistical sequences and then studies on their many properties or how to use them, too this idea should be also extended to summability theory with consequences from our definitions about these notions defined through a topology similar like g-bounds are one suggestion et cetera consequently several function classes raised at section can be proper example applied as scrutiny. In this paper, pointwise and uniform statistical convergence was studied.

4. REFERENCES

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