

# APPLICATION OF SCHEFFE'S (5,2) OPTIMIZATION MODEL IN THE FLEXURAL AND SPLIT TENSILE STRENGTHS DETERMINATION OF NYLON FIBRE REINFORCED CONCRETE [NFRC]

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## ABSTRACT

This research work is aimed at using Scheffe's Second Degree Model for five component mixture to optimize the Flexural Strength [FS] and Split Tensile Strength [STS] of Nylon Fibre Reinforced Concrete [NFRC]. Using Scheffe's Simplex method, the Flexural Strength and Split Tensile Strength of NFRC were determined for different mix proportions. Control experiments were also carried out and the flexural and split tensile strengths evaluated. The result of the student's t-test shows that the strengths predicted by the model and the corresponding experimentally observed results are correlated. Maximum design strengths recorded for the flexural test at 14 and 28 days were 6.00MPa and 7.44MPa respectively, while those recorded for the splitting tensile test were 3.98MPa and 5.26MPa respectively. Thus, NFRC controllable design strengths values are capable of sustaining construction of light-weight and heavy weight structures such as construction of Bridge, Building pillars, Sidewalks, Building floors, Drainage pipes, Septic tanks etc., still satisfying all the required economic, aesthetic and safety criteria.

**Keywords:** Optimisation, Scheffe's (5,2) Polynomial Model, NFRC, Flexural Strength, Split Tensile Strength, Mixture Design

## 1. INTRODUCTION

Generally, concrete is strong in compression and weak in tension. Despite the fact that conventional steel reinforcement are usually provided to address the latter in a typical reinforced concrete member, the knowledge of tensile and split tensile strengths of concrete is also of utmost importance to the concrete designer. Again, as the goal of every engineering design is to ensure that issues that are related to safety, economy and aesthetics are carefully addressed, researches towards achieving the goals are always on course. As a result, concrete, as the most widely used material in the construction industries has been undergoing changes both as a material and due to technological advancement. By definition, according to Oyenuga (2008), concrete is a composite inert material comprising of a binder course (cement), mineral filler or aggregates and water. Concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Furthermore, according to Neville (1990), concrete, plays an important part in all building structures owing to its numerous advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. One of the research output has seen the expensive conventional reinforced bars being substituted by less expensive fibres. This type of research is known as Fibre reinforced concrete (FRC) research. In general, FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete as well as uniformly dispersed fibre. The main purposes of incorporating the fibrous materials remain to increase the concrete's durability and structural integrity and at the same time save costs. The last purpose, cost efficiency is achievable because, all fibres reduce the concrete's need for steel reinforcements. And since fibre reinforcement tends to be less expensive than steel bars (and less likely to corrode), it makes FRC more cost-effective. Nylon Fibre Reinforced Concrete (NFRC) is one form of FRC and is the concrete mixture where the conventionally steel reinforcement in concrete production is replaced with nylon fibre. It is important to note that one of the ultimate aims of studying the various properties of the materials of concrete is to enable concrete technologist to design a concrete mix for a particular strength and durability (Shetty, 2006). For this present work, special properties of NFRC under investigation are the flexural strength and the split tensile strength. By definition, flexural strength (modulus of rupture) is the ability of the material to withstand bending forces applied perpendicular to its longitudinal axis. It is also defined as the maximum bending stress that can be applied to the material before it yields. On the other hand, splitting tensile strength test on concrete cylinder is a method to determine the tensile strength of concrete. It is generally carried out to obtain the tensile strength of concrete, and the stress field in the tests is actually a biaxial stress field with compressive stress three times greater than the tensile stress. The split tensile strength test is an indirect method of testing tensile strength of concrete and is generally greater than direct tensile strength and lower than flexural strength.

By definition, an optimization problem is one requiring the determination of the optimal (maximum or minimum) value of a given function, called the objective function, subject to a set of stated restrictions, or constraints placed on the variables concerned. The objective of mix design, according to Shacklock (1974), is to determine the most appropriate mixes in which to use the constituent materials to meet the needs of construction work. Specifically, optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the components is lowest, yet satisfying the required performance of concrete, such as strength, workability and durability, flexibility, etc. According to Jackson and Dhira (1996), concrete mix design remains the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. From the above definition, the cost of any concrete should include, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. In the context of the above guidelines, the empirical mix design methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) are a little bit complex as well as time consuming. This is because, they involve a lot of trial mixes and complex statistical computations before the desired strength of the concrete can be obtained. Thus, optimization of the concrete mixture design is more preferable, as it is the fastest method, more result oriented best option, most convenient and the most efficient way of selecting concrete mix ratios or proportions for better efficiency and better performance of concrete. A typical example of optimization model is Scheffe's Model. It could be in the form of Scheffe's Second Degree Model or Scheffe's Third Degree Model. In this present study, Scheffe's Second Degree Model for five components mixtures (namely Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Nylon Fibre) is presented.

This present study examines the application of Scheffe's Second Degree Model for five component mixture in the optimization of the Flexural Strength and Split Tensile Strength of NFRC. Although, some related works have been done by many researchers, none has been able to address the real subject matter. For instance, Ganesh Kumar and others (2019) have carried out a study on waste nylon fibre in concrete. Samrose and Mutsuddy (2019) have investigated the durability of NFRC. Hossain and others (2012) have also investigated the effect of NF in concrete rehabilitation. Ali and others (2018) have carried out a study on NFRC through partial replacement of cement with metakaolin. Song and others (2005) also investigated the strength properties of NFRC and PFRC respectively. Hassan and others (2022) investigated the Mechanical Properties and Absorption of High-Strength Fiber-Reinforced Concrete (HSFRC) with Sustainable Natural Fibers. In recent years, many researchers have used Scheffe's method to carry out one form of optimization work or the other. Nwakonobi and Osadebe (2008) for instance, used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Ezech and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was employed by Ezech and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezech and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. Mbadike and Osadebe (2013) applied Scheffe's (4,2) model to optimize the compressive strength of Laterite Concrete. Egamana and Sule (2017) carried out an optimization work on the compressive strength of periwinkle shell aggregate concrete. Ibearugbulem (2006) and Okere (2006) used Scheffe's mathematical model in the optimization of compressive strength of Periwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3). Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation, Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid-Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With

Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC). Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2023c) applied Scheffe's (6,2) model to optimize the Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC). Finally, Nwachukwu and others (2023d) made use of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC). Nwachukwu and others (2023e) used optimization techniques in the Flexural Strength And Split Tensile Strength determination of Hybrid Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023f) applied Scheffe's Optimization model in the evaluation of Flexural Strength And Split Tensile Strength Of Plastic Fibre Reinforced Concrete (PLFRC). Nwachukwu and Opara (2023) in their paper presented at the Conference Proceedings of the Nigeria Society of Engineers, demonstrated the use of Snail Shells Ash (SSA) in the partial replacement of cement using Scheffe's (5,2) optimization model. Nwachukwu and others (2024a) applied the use of Scheffe's (6,2) model to evaluate the optimum flexural and split tensile strengths of Periwinkle Shells Ash (PSA)- Mussel Shells Ash (MSA)- Cement Concrete (PMCC). Nwachukwu and others (2024b) applied the use of Scheffe's (6,2) model to evaluate the optimum compressive strength of Periwinkle Shells Ash (PSA)- Snail Shells Ash (SSA)- Cement Concrete (PSCC). Nwachukwu and others (2024c) applied Scheffe's (5,2) model to evaluate the compressive strength of Plastic Fibre Reinforced Concrete [PLFRC]. Nwachukwu and others (2024d) applied the use of Scheffe's Third Degree Model to optimize the compressive strength of HPNFRC. Finally, Nwachukwu and others (2024e) applied the use of Scheffe's Third Degree Regression Model to optimize the compressive strength of MSFRC. From the works reviewed so far, it can be envisaged that the subject matter has not been fully addressed as it is obvious that no work has been done on the use of Scheffe's Second Degree Model to optimize the flexural strength and split tensile strength of NFRC. Thus, there is urgent need for this present research work.

## 2. METHODOLOGY

### 2.1 MATERIALS FOR NFRC MIXTURES

In this present study, the materials under investigation are cement, water, fine and coarse aggregate and nylon fibre. The cement is Dangote cement, a brand of Ordinary Portland Cement, conforming to British Standard Institution BS 12 (1978). The water is procured from potable water from the available clean water source and was applied in accordance with ASTM C1602/C1602M-22 (2022). The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. Both fine and coarse aggregates were procured and prepared in accordance with ASTM C33/C33M-18 (2018). The same size and nature of nylon fibre used previously by Nwachukwu and others (2022d) in the compressive strength investigation, is the same as the one being used in this present work based on Scheffe's second degree model.

### 2.2. BASIC FACTS ON NFRC SCHEFFE'S (5,2) OPTIMIZATION THEORY

As a simplex lattice is defined as a structural representation of lines joining the atoms of a mixture, it is important to note that these atoms are the constituent components of the same mixture. For instance, when considering this present NFRC concrete mixture, the five constituent elements are Water, Cement, Fine Aggregate, Coarse Aggregate and Nylon Fibre. One basic fact to know, according to Obam (2009) is that the mixture components are usually subject to the constraint that the sum of all the components must be equal to 1 as stated in Eqn. (1):  $X_1 + X_2 + X_3 + \dots + X_q = 1$  ;  $\Rightarrow \sum_{i=1}^q X_i = 1$  (1)

where  $X_i \geq 0$  and  $i = 1, 2, 3 \dots q$ , and  $q$  = the number of mixtures.

#### 2.2.1. EVALUATION OF DESIGN POINTS FOR NFRC SCHEFFE'S (2, 2) MIXTURES

In general, the (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains  $q^{m-1}C_m$  points where each components proportion takes (m+1) equally spaced values  $X_i =$

$0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; i = 1, 2, \dots, q$  ranging between 0 and 1 and all possible mixture with these component proportions are used, and  $m$  is scheffe's polynomial degree, which in this present study is 2.

For example a (3, 2) lattice consists of  $^{3+2-1}C_2$  i.e.  $^4C_2 = 6$  points. Each  $X_i$  can take  $m+1 = 3$  possible values; that is  $x = 0, \frac{1}{2}, 1$  with which the possible design points are:  $(1, 0, 0), (0, 1, 0), (0, 0, 1), (\frac{1}{2}, \frac{1}{2}, 0), (0, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 0, \frac{1}{2})$ . In order to determine the number of coefficients/terms/ design points required for a given Scheffe's component mixtures, the following general formula is applied:  $k = \frac{(q+m-1)!}{(q-1)! \cdot m!}$  Or  $^{q+m-1}C_m$  **2(a-b)** Where  $k =$  number of coefficients/ terms / design points,  $q =$  number of components = 5 in this work and  $m =$  number of degree of polynomial = 2 in this present work. Using either of Eqn. (2),  $k_{(5,2)} = 15$ . Consequently, the possible design points for Scheffe's (5,2) lattice can be as follows:

$A_1 (1, 0, 0, 0, 0); A_2 (0, 1, 0, 0, 0); A_3 (0, 0, 1, 0, 0); A_4 (0, 0, 0, 1, 0); A_5 (0, 0, 0, 0, 1); A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0, 0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) \text{ and } A_{45} (0, 0, 0, 0.5, 0.5)$  **(3)**

According to Obam (2009), a Scheffe's polynomial function of degree,  $m$  in the  $q$  variable  $X_1, X_2, X_3, X_4 \dots X_q$  is given in the form of Eqn.(4):  $P = b_0 + \sum b_i x_i + \sum b_{ij} x_j + \sum b_{ijk} x_j x_k + \dots + \sum b_{i_1 i_2 \dots i_m} x_{i_1} x_{i_2} \dots x_{i_m}$  **(4)**

where  $(1 \leq i \leq q, 1 \leq i \leq j \leq k \leq q, 1 \leq i_1 \leq i_2 \leq \dots \leq i_m \leq q \text{ respectively})$ ,  $b =$  constant coefficients and  $P$  is the response (the response is a polynomial function of pseudo component of the mix) which represents the property under study, which, in this case is the Flexural Strength ( $P^F$ ) or Split Tensile Strength ( $P^S$ ) as the case may be.

As this research work is based on the (5, 2) simplex, the actual form of Eqn. (4) has already been developed by Nwachukwu and others (2017) and will be applied subsequently

### 2.2.2. PSEUDO AND ACTUAL COMPONENTS IN NFRC SCHEFFE'S (5,2) MIX DESIGN

In Scheffe's mix design, the relationship between the actual components and the pseudo components has been established as :  $Z = A * X$  **(5)**

where  $Z$  is the actual component;  $X$  is the pseudo component and  $A$  is the coefficient of the relationship

Re-arranging Eqn. (5) yields:  $X = A^{-1} * Z$  **(6)**

### 2.2.3. FORMULATION OF REGRESSION EQUATION FOR NFRC SCHEFFE'S (5, 2) LATTICE

The Regression equation by Scheffe (1958), describing the response is given in Eqn.(4). But, for Scheffe's (5,2) simplex lattice, the polynomial equation for five component mixtures has been derived from Eqn.(4) by Nwachukwu and others (2017). Eqn.(7) gives the simplified version :

$P = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5$  **(7)**

### 2.2.4. COEFFICIENTS DETERMINATION OF THE NFRC SCHEFFE'S (5, 2) POLYNOMIAL

From the work of Nwachukwu and others (2022h), the simplified equations for the coefficients of the Scheffe's (5, 2) polynomial are expressed as follows. :

$\beta_1 = P_1; \beta_2 = P_2; \beta_3 = P_3; \beta_4 = P_4; \beta_5 = P_5; \beta_{12} = 4P_{12} - 2P_1 - 2P_2; \beta_{13} = 4P_{13} - 2P_1 - 2P_3;$  **8(a-g)**

$\beta_{14} = 4P_{14} - 2P_1 - 2P_4; \beta_{15} = 4P_{15} - 2P_1 - 2P_5; \beta_{23} = 4P_{23} - 2P_2 - 2P_3; \beta_{24} = 4P_{24} - 2P_2 - 2P_4;$  **9(a-d)**

$\beta_{25} = 4P_{25} - 2P_2 - 2P_5; \beta_{34} = 4P_{34} - 2P_3 - 2P_4; \beta_{35} = 4P_{35} - 2P_3 - 2P_5; \beta_{45} = 4P_{45} - 2P_4 - 2P_5$  **10(a-d)**

Where  $P_i =$  Response Function (Flexural Strength or Split Tensile Strength as the case may be ) for the pure component,  $i$

### 2.2.5. MIXTURE DESIGN MODEL FOR NFRC

When we substitute Eqns. (8)-(10) into Eqn. (7), we obtain the mixture design model for the NFRC mixture based on Scheffe's (5, 2) lattice for the flexural and split tensile strengths.

### 2.2.6. EVALUATION OF ACTUAL AND PSEUDO MIX RATIOS FOR THE NFRC SCHEFFE'S (5, 2) DESIGN LATTICE

#### A. AT THE NFRC INITIAL EXPERIMENTAL TEST POINTS [IETP]

As confirmed from the practical knowledge, the requirement of simplex lattice design from Eqn.(1) makes it impossible to use the conventional mix ratios such as 1:2:4, 1:1.3:6, as the case may be., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions were chosen for the five points/vertices.:



$$A_1 (0.67:1: 1.7: 2:0.5); A_2 (0.56:1:1.6:1.8:0.8); A_3 (0.5:1:1.2:1.7:1); A_4 (0.7:1:1:1.8:1.2) \\ \text{And } A_5 (0.75:1:1.3:1.2:1.5) \quad (11)$$

which represent water/cement ratio, cement, fine aggregate, coarse aggregate and nylon fibre.

For the pseudo mix ratio, we have the following corresponding mix ratios at the vertexes:

$$A_1(1:0:0:0:0), A_2(0:1:0:0:0), A_3(0:0:1:0:0), A_4(0:0:0:1:0) \text{ and } A_5(0:0:0:0:1) \quad (12)$$

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns.(5) and (6) are used..

Substituting the mix ratios from point  $A_1$  into Eqn. (5) we have:

$$\begin{matrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \end{matrix} \left\{ \begin{matrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{matrix} \right\} \begin{matrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \quad (13)$$

Transforming the R.H matrix and solving, we obtain:

$$\begin{matrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{matrix} \left\{ \begin{matrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{matrix} \right\} \begin{matrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{matrix} \quad (14)$$

$$\begin{matrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{matrix} \left\{ \begin{matrix} 3.99 & 10.37 & -2.14 & -3.05 & -4.62 \\ -4.88 & -21.46 & 5.40 & 5.95 & 7.31 \\ -1.78 & 17.83 & -3.49 & -4.20 & -4.62 \\ 1.04 & -9.24 & 0.37 & 3.28 & 2.69 \\ 1.63 & 3.49 & -0.13 & -1.98 & -0.77 \end{matrix} \right\} \begin{matrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{matrix} \quad (15)$$

Considering the mix ratios at the midpoints from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn. (15) will give the corresponding actual mix ratio.

For instance, at point  $A_{12}$

$$\begin{matrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{matrix} \left\{ \begin{matrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{matrix} \right\} \begin{matrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{matrix} = \begin{matrix} 0.62 \\ 1 \\ 1.65 \\ 1.90 \\ 0.6 \end{matrix} \quad (16)$$

Solving, we have :  $Z_1 = 0.62, Z_2 = 1, Z_3 = 1.65, Z_4 = 1.9, Z_5 = 0.65$ . The rest are shown in Table 1

To generate the polynomial coefficients, fifteen experimental tests (each for Flexural Strength and Split Tensile Strength) will be carried out and the corresponding mix ratio is as depicted in Table 1.

**Table 1:** Pseudo (X) and Actual (Z) Mix Ratio For NFRC Based On Scheffe's (5,2) Lattice For IETP (For Flexural Strength And Split Tensile Strength).

S/N	IETP	PSEUDO COMPONENT					RESPONSE SYMBOL	ACTUAL COMPONENT				
		$X_1$	$X_2$	$X_3$	$X_4$	$X_5$		$Z_1$	$Z_2$	$Z_3$	$Z_4$	$Z_5$
1	$E_1$	1	0	0	0	0	$P_1$	0.67	1	1.70	2.00	0.50
2	$E_2$	0	1	0	0	0	$P_2$	0.56	1	1.60	1.80	0.80
3	$E_3$	0	0	1	0	0	$P_3$	0.50	1	1.20	1.70	1.00
4	$E_4$	0	0	0	1	0	$P_4$	0.70	1	1.00	1.80	1.20
5	$E_5$	0	0	0	0	1	$P_5$	0.75	1	1.30	1.20	1.50
6	$E_{12}$	0.50	0.50	0	0	0	$P_{12}$	0.62	1	1.65	1.90	0.65
7	$E_{13}$	0.50	0	0.50	0	0	$P_{13}$	0.59	1	1.45	1.85	0.75
8	$E_{14}$	0.50	0	0	0.50	0	$P_{14}$	0.69	1	1.35	1.90	0.85

9	E <sub>15</sub>	0.50	0	0	0	0.50	P <sub>15</sub>	0.71	1	1.50	1.60	1.00
10	E <sub>23</sub>	0	0.50	0.50	0	0	P <sub>23</sub>	0.53	1	1.40	1.75	0.90
11	E <sub>24</sub>	0	0.50	0	0.50	0	P <sub>24</sub>	0.63	1	1.30	1.80	1.00
12	E <sub>25</sub>	0	0.50	0	0	0.50	P <sub>25</sub>	0.66	1	1.45	1.50	1.15
13	E <sub>34</sub>	0	0	0.50	0.50	0	P <sub>34</sub>	0.60	1	1.10	1.75	1.10
14	E <sub>35</sub>	0	0	0.50	0	0.5	P <sub>35</sub>	0.63	1	1.25	1.45	1.25
15	E <sub>45</sub>	0	0	0	0.5	0.5	P <sub>45</sub>	0.73	1	1.15	1.50	1.50

#### B. AT THE NFRC EXPERIMENTAL (CONTROL) TEST POINTS [ECTP]

For the purpose of this research, fifteen different controls test (each for Flexural Strength and Split Tensile Strength) were predicted which according to Scheffes, their summation should not be more than one. Thus, the following pseudo mix proportions are applicable at the control points:

C<sub>1</sub> (0.25, 0.25, 0.25, 0.25, 0), C<sub>2</sub> (0.25, 0.25, 0.25, 0, 0.25), C<sub>3</sub> (0.25, 0.25, 0, 0.25, 0.25), C<sub>4</sub> (0.25, 0, 0.25, 0.25, 0.25), C<sub>5</sub> (0, 0.25, 0.25, 0.25, 0.25), C<sub>12</sub> (0.20, 0.20, 0.20, 0.20, 0.20), C<sub>13</sub> (0.30, 0.30, 0.30, 0.10, 0), C<sub>14</sub> (0.30, 0.30, 0.30, 0, 0.10), C<sub>15</sub> (0.30, 0.30, 0, 0.30, 0.1), C<sub>23</sub> (0.30, 0, 0.30, 0.30, 0.1), C<sub>24</sub> (0, 0.30, 0.30, 0.30, 0.10), C<sub>25</sub> (0.10, 0.30, 0.30, 0.30, 0), C<sub>34</sub> (0.30, 0.10, 0.30, 0.30, 0), C<sub>35</sub> (0.30, 0.30, 0.10, 0.30, 0), C<sub>45</sub> (0.10, 0.20, 0.30, 0.40, 0), (17)

Substituting into Eqn.(16) , we obtain the values of the actual mixes as follows:

#### Control 1 C<sub>1</sub>

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{bmatrix} \begin{Bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.61 \\ 1 \\ 1.38 \\ 1.8 \\ 0.5 \end{Bmatrix} \quad (18)$$

The rest are shown in Table 2

**Table 2:** Actual & Pseudo Component Of NFRC Based On Scheffe 's (5,2) Lattice For ECTP

S/ N	ECTP	PSEUDO COMPONENTS					RESPONSE SYMBOL	ACTUAL COMPONENTS				
		Wa (X <sub>1</sub> )	Cem (X <sub>1</sub> )	FA (X <sub>3</sub> )	CA (X <sub>4</sub> )	NF (X <sub>5</sub> )		Water (Z <sub>1</sub> )	Cem (Z <sub>2</sub> )	FA (Z <sub>3</sub> )	CA (Z <sub>4</sub> )	NF (Z <sub>5</sub> )
1	C <sub>1</sub>	0.25	0.25	0.25	0.25	0.00	P <sub>1</sub>	0.61	1	1.38	1.83	0.50
2	C <sub>2</sub>	0.25	0.25	0.25	0.00	0.25	P <sub>2</sub>	0.62	1	1.45	1.68	0.80
3	C <sub>3</sub>	0.25	0.25	0.00	0.25	0.25	P <sub>3</sub>	0.67	1	1.40	1.70	1.00
4	C <sub>4</sub>	0.25	0.00	0.25	0.25	0.25	P <sub>4</sub>	0.66	1	1.30	1.68	1.20
5	C <sub>5</sub>	0.00	0.25	0.25	0.25	0.25	P <sub>5</sub>	0.63	1	1.28	1.63	1.50
6	C <sub>12</sub>	0.20	0.20	0.20	0.20	0.20	P <sub>12</sub>	0.64	1	1.36	1.70	0.65
7	C <sub>13</sub>	0.30	0.30	0.30	0.10	0.00	P <sub>13</sub>	0.59	1	1.45	1.83	0.75
8	C <sub>14</sub>	0.30	0.30	0.30	0.00	0.10	P <sub>14</sub>	0.59	1	1.48	1.77	0.85
9	C <sub>15</sub>	0.30	0.30	0.00	0.30	0.10	P <sub>15</sub>	0.65	1	1.42	1.80	1.00
10	C <sub>23</sub>	0.30	0.00	0.30	0.30	0.10	P <sub>23</sub>	0.64	1	1.30	1.77	0.90
11	C <sub>24</sub>	0.00	0.30	0.30	0.30	0.10	P <sub>24</sub>	0.60	1	1.27	1.71	1.00
12	C <sub>25</sub>	0.10	0.30	0.30	0.30	0.00	P <sub>25</sub>	0.60	1	1.31	1.79	1.15
13	C <sub>34</sub>	0.30	0.10	0.30	0.30	0.00	P <sub>34</sub>	0.62	1	1.33	1.83	1.10
14	C <sub>35</sub>	0.30	0.30	0.10	0.30	0.00	P <sub>35</sub>	0.63	1	1.41	1.85	1.25
15	C <sub>45</sub>	0.10	0.20	0.30	0.40	0.00	P <sub>45</sub>	0.61	1	1.25	1.79	0.50

### 2.2.7. MEASUREMENT OF QUANTITIES OF NFRC MATERIALS

The actual component as transformed from Eqn. (17), Tables (1) and (2) were used to measure out the quantities of Water/Cement Ratio ( $Z_1$ ), Cement ( $Z_2$ ), Fine Aggregate ( $Z_3$ ), Coarse Aggregate ( $Z_4$ ), and Nylon Fibre ( $Z_5$ ) using a weighing balance of 50kg capacity in their respective ratios for the eventual Concrete Beam Cube and Concrete Cylindrical specimen at the laboratory.

Mathematically, Measured Quantity,  $M^Q$  of NFRC Mixture is given by Eqn.(19)

$$M^Q = \frac{X}{T} * Y \quad (19) \quad \text{Where, } X$$

= Individual mix ratio at each test point = 0.67 for  $Z_1$  at  $E_1$  in Table 1, for example.

T = Sum of mix ratios at each test point = 5.87 at  $E_1$  in Table 1, for example

And Y = Average weight of Concrete cube/beam/cylinder

For the Flexural Strength concrete beam mould of 15cm\*15cm\*60cm, Average Y from experience = 30kg

For the Split Tensile Strength Concrete cylinder mould of 15cm\*30cm, Average Y from experience =12.5kg

Samples of measured quantities can be seen from the works of Nwachukwu and others 2024 (a and b).

## 2.3. METHOD

### 2.3.1. METHODS FOR NFRC FLEXURAL STRENGTH TEST

**A. NFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR FLEXURAL STRENGTH TEST** In this experimental investigation, the standard size of specimen (mould) for the Flexural Strength measures 150mm\*150mm\*600mm. The mould is of steel metal with sufficient thickness to prevent spreading or warping. The mould is constructed with the longer dimension horizontal and in such a manner as to facilitate the removal of the moulded specimen without damage. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cubes. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (10). Twenty-four (24) hours after moulding, curing commenced. Test specimens are stored in water at a temperature of 24<sup>0</sup> to 30<sup>0</sup> for 48 hours before testing. They are tested immediately on removal from the water whilst they are still in a wet condition. After 14 and 28 days of curing respectively, the specimens were taken out of the curing tank for flexural strength determination.

### B. NFRC FLEXURAL STRENGTH TEST PROCEDURE/CALCULATION

Flexural strength testing was done in accordance with BS 1881 – part 118 (1983) - Method of determination of Flexural Strength, ASTM C78/C78M-22 (2022) and ACI (1989) guideline. In this present study, two samples were crushed for each mix ratio. In each case, the Flexural Strength of each specimen/sample which is expressed as the Modulus of Rupture (MOR) was then calculated to the nearest 0.05 MPa using Eqn.(20)

$$MOR = \frac{PL}{bd^2} \quad (20)$$

where b = measured width in cm of the specimen, d = measured depth in cm of the specimen at the point of failure, where L = Length in cm of the span on which the specimen was supported and P = maximum load in kg applied to the specimen.

### 2.3.2. METHODS FOR SPLIT TENSILE STRENGTH TEST

#### A. NFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR SPLIT TENSILE STRENGTH TEST

The specimen for the Split Tensile Strength is Concrete Cylindrical specimen measuring diameter 150 mm and length 300 mm. They were cast with plastic fibres and the specimen was loaded for ultimate compressive load under Universal Testing Machine (UTM) for each mix. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cubes. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (10).. After 14 and 28 days of curing respectively, the specimens were taken out of the curing tank for the Split Tensile Strength determination.

#### B. NFRC SPLIT STRENGTH TEST PROCEDURE/CALCULATION

The cylindrical split tensile test was done using the universal testing machine in accordance with BS

EN 12390-6:2009 and ASTM C 496/ C 496 M-11. Two samples were crushed for each mix proportion and in each case, the Split Tensile Strength of each specimen/sample was then calculated using Eqn. (21)

$$F_t = \frac{2P}{\pi D L} \quad (21)$$

$\pi D L$

Where,  $F_t$  = Split Tensile Strength, MPa ,  $P$  = maximum applied load (that is Load at failure, N) ;  $D$  = diameter of the cylindrical specimen (Dia. Of cylinder, mm); and  $L$  = Length of the specimen (Length of cylinder, mm).

### 3. RESULTS PRESENTATION AND DISCUSSION

#### 3.1. NFRC RESPONSES (FLEXURAL STRENGTH AND SLIT TENSILE STRENGTH) FOR THE IETP

The results of the Responses (Flexural Strength and Split Tensile Strength) test based on Eqns.(20 and 21) are shown in Table 3.

**Table 3:** 14<sup>TH</sup> AND 28<sup>TH</sup> NFRC Responses (Flexural Strength, FS and Split Tensile Strength, STS) Test Results From IETP Based on Eqns.(20and 21).

S/N	IETP	EXPT. NO.	14 <sup>TH</sup> DAY RESPONSE P <sub>t</sub> , MPa		28 <sup>TH</sup> DAY RESPONSE P <sub>t</sub> , MPa		RESPONSE SYMBOL	14 <sup>TH</sup> DAY AVERAGE RESPONSE P, MPa		28 <sup>TH</sup> DAY AVERAGE RESPONSE, P, MPa	
			FS	STS	FS	STS		FS	STS	FS	STS
1	E <sub>1</sub>	NFRC/ E <sub>1</sub> A	4.07	2.63	4.12	3.34	P <sub>1</sub>	4.10	2.65	4.18	3.36
		NFRC/ E <sub>1</sub> B	4.12	2.67	4.24	3.38					
2	E <sub>2</sub>	NFRC/ E <sub>2</sub> A	3.98	2.98	4.72	3.23	P <sub>2</sub>	3.96	2.99	4.74	3.25
		NFRC/ E <sub>2</sub> B	3.94	3.00	4.75	3.26					
3	E <sub>3</sub>	NFRC/ E <sub>3</sub> A	5.43	3.43	4.84	3.54	P <sub>3</sub>	5.49	3.46	4.81	3.57
		NFRC/ E <sub>3</sub> B	5.55	3.49	4.78	3.59					
4	E <sub>4</sub>	NFRC/ E <sub>4</sub> A	5.98	4.00	7.46	5.28	P <sub>4</sub>	6.00	3.98	7.44	5.26
		NFRC/ E <sub>4</sub> B	6.02	3.96	7.42	5.24					
5	E <sub>5</sub>	NFRC/ E <sub>5</sub> A	5.88	3.56	7.00	5.12	P <sub>5</sub>	5.87	3.58	7.06	5.15
		NFRC/ E <sub>5</sub> B	5.86	3.59	7.11	5.18					
6	E <sub>12</sub>	NFRC/ E <sub>12</sub> A	3.82	2.52	4.06	3.12	P <sub>12</sub>	3.80	2.50	4.08	3.08
		NFRC/ E <sub>12</sub> B	3.78	2.48	4.10	3.04					
7	E <sub>13</sub>	NFRC/ E <sub>13</sub> A	5.14	3.04	4.48	4.44	P <sub>13</sub>	5.16	3.06	4.50	4.43
		NFRC/ E <sub>13</sub> B	5.18	3.08	4.52	4.42					
8	E <sub>14</sub>	NFRC/ E <sub>14</sub> A	4.74	3.45	4.88	4.28	P <sub>14</sub>	4.78	3.50	4.89	4.27
		NFRC/ E <sub>14</sub> B	4.82	3.54	4.90	4.26					



		E <sub>14</sub> B									
9	E <sub>15</sub>	NFRC/ E <sub>15</sub> A	5.08	2.98	6.75	3.86	P <sub>15</sub>	5.11	2.97	6.77	3.86
		NFRC/ E <sub>15</sub> B	5.13	2.96	6.78	3.85					
10	E <sub>23</sub>	NFRC/ E <sub>23</sub> A	5.45	2.76	6.54	4.32	P <sub>23</sub>	5.48	2.78	6.55	4.34
		NFRC/ E <sub>23</sub> B	5.51	2.80	6.56	4.36					
11	E <sub>24</sub>	NFRC/ E <sub>24</sub> A	4.86	2.84	6.43	4.56	P <sub>24</sub>	4.82	2.86	6.45	4.59
		NFRC/ E <sub>24</sub> B	4.78	2.88	6.47	4.61					
12	E <sub>25</sub>	NFRC/ E <sub>25</sub> A	4.23	3.02	5.34	4.34	P <sub>25</sub>	4.26	3.05	5.38	4.38
		NFRC/ E <sub>25</sub> B	4.28	3.08	5.42	4.42					
13	E <sub>34</sub>	NFRC/ E <sub>34</sub> A	4.53	3.45	4.65	3.87	P <sub>34</sub>	4.56	3.49	4.67	3.89
		NFRC/ E <sub>34</sub> B	4.58	3.52	4.68	3.90					
14	E <sub>35</sub>	NFRC/ E <sub>35</sub> A	5.28	3.72	6.34	3.68	P <sub>35</sub>	5.31	3.74	6.38	3.69
		NFRC/ E <sub>35</sub> B	5.34	3.75	6.42	3.70					
15	E <sub>45</sub>	NFRC/ E <sub>45</sub> A	4.82	3.86	5.76	3.86	P <sub>45</sub>	4.84	3.83	5.81	3.89
		NFRC/ E <sub>45</sub> B	4.86	3.79	5.86	3.92					

### 3.2. NFRC RESPONSES (FLEXURAL STRENGTH AND SPLIT TENSILE STRENGTH) FOR THE ECTP

The responses (Flexural strength & Split Tensile Strength) from experimental (control) tests are shown in Table 4

**Table 4:** NFRC Scheffe's (5,2) Responses (Flexural strength and Split Tensile Strength) From ECTP

S/ N	ECT P	EXPT NO	14 <sup>TH</sup> DAY RESPONS E (MPa)		28 <sup>TH</sup> DAY RESPONS E (MPa)		Z <sub>1</sub>	Z <sub>2</sub>	Z <sub>3</sub>	Z <sub>4</sub>	Z <sub>5</sub>	14 <sup>TH</sup> DAY AVERAGE RESPONS E (MPa)		28 <sup>TH</sup> DAY AVERAGE RESPONS E (MPa)	
			FS	STS	FS	STS						FS	STS	FS	STS
1	C <sub>1</sub>	NFRC / C <sub>1</sub> A	4.01	2.61	4.09	3.36	0.6 1	1	1.3 8	1.8 3	0.5 0	4.05	2.62	4.11	3.3 7
		NFRC / C <sub>1</sub> B	4.08	2.62	4.12	3.38									

		C <sub>1</sub> B													
<b>2</b>	C <sub>2</sub>	NFRC / C <sub>2</sub> A	3.91	2.92	4.71	3.32	0.6 2	1	1.4 5	1.6 8	0.8 0	3.91	2.91	4.72	3.3 3
		NFRC / C <sub>2</sub> B	3.90	2.89	4.72	3.34									
<b>3</b>	C <sub>3</sub>	NFRC / C <sub>3</sub> A	5.37	3.41	4.81	3.47	0.6 7	1	1.4 0	1.7 0	1.0 0	5.39	3.42	4.80	4.4 8
		NFRC / C <sub>3</sub> B	5.41	3.42	4.79	3.49									
<b>4</b>	C <sub>4</sub>	NFRC / C <sub>4</sub> A	5.90	4.00	7.41	5.22	0.6 6	1	1.3 0	1.6 8	1.2 0	5.95	4.00	7.41	5.2 6
		NFRC / C <sub>4</sub> B	6.00	3.99	7.40	5.29									
<b>5</b>	C <sub>5</sub>	NFRC / C <sub>5</sub> A	5.84	3.51	7.12	5.17	0.6 3	1	1.2 8	1.6 3	1.5 0	5.83	3.51	7.17	5.1 8
		NFRC / C <sub>5</sub> B	5.81	3.51	7.21	5.18									
<b>6</b>	C <sub>12</sub>	NFRC / C <sub>12</sub> A	3.80	2.47	4.04	3.15	0.6 4	1	1.3 6	1.7 0	0.6 5	3.78	2.50	4.06	3.1 7
		NFRC / C <sub>12</sub> B	3.76	2.52	4.07	3.18									
<b>7</b>	C <sub>13</sub>	NFRC / C <sub>13</sub> A	5.11	3.08	4.41	4.31	0.5 9	1	1.4 5	1.8 3	0.7 5	5.12	3.09	4.42	4.3 3
		NFRC / C <sub>13</sub> B	5.12	3.09	4.43	4.34									
<b>8</b>	C <sub>14</sub>	NFRC / C <sub>14</sub> A	4.71	3.41	4.81	4.28	0.5 9	1	1.4 8	1.7 7	0.8 5	4.75	3.41	4.82	4.2 9
		NFRC / C <sub>14</sub> B	4.78	3.42	4.83	4.29									
<b>9</b>	C <sub>15</sub>	NFRC / C <sub>15</sub> A	5.00	2.98	6.71	3.88	0.6 5	1	1.4 2	1.8 0	1.0 0	5.02	2.96	6.72	3.8 4
		NFRC / C <sub>15</sub> B	5.04	2.94	6.72	3.80									
<b>10</b>	C <sub>23</sub>	NFRC / C <sub>23</sub> A	5.42	2.72	6.51	4.39	0.6 4	1	1.3 0	1.7 7	0.9 0	5.42	2.73	6.52	4.3 8
		NFRC / C <sub>23</sub> B	5.41	2.73	6.52	4.37									

11	C <sub>24</sub>	NFRC / C <sub>24</sub> A	4.80	2.81	6.47	4.59	0.6 0	1	1.2 7	1.7 1	1.0 0	4.81	2.82	6.48	4.5 7
		NFRC / C <sub>24</sub> B	4.81	2.82	6.49	4.55									
12	C <sub>25</sub>	NFRC / C <sub>25</sub> A	4.21	3.00	5.38	4.39	0.6 0	1	1.3 1	1.7 9	1.1 5	4.21	3.01	5.42	4.3 6
		NFRC / C <sub>25</sub> B	4.22	3.02	5.46	4.33									
13	C <sub>34</sub>	NFRC / E <sub>34</sub> A	4.51	3.42	4.61	3.42	0.6 2	1	1.3 3	1.8 3	1.1 0	4.52	3.45	9.23	3.4 3
		NFRC / E <sub>34</sub> B	4.52	3.47	4.64	3.43									
14	C <sub>35</sub>	NFRC / E <sub>35</sub> A	5.22	3.67	6.37	3.73	0.6 3	1	1.4 1	1.8 5	1.2 5	5.23	3.68	6.35	3.7 4
		NFRC / E <sub>35</sub> B	5.23	3.68	6.32	3.74									
15	C <sub>45</sub>	NFRC / E <sub>45</sub> A	4.78	3.81	5.73	3.65	0.6 1	1	1.2 5	1.7 9	0.5 0	4.77	3.82	5.75	3.6 9
		NFRC / E <sub>45</sub> B	4.76	3.82	5.76	3.72									

### 3.3. SCHEFFE' S (5,2) POLYNOMIAL MODEL FOR THE NFRC RESPONSES (FLEXURAL STRENGTH AND SPLIT TENSILE STRENGTH) AT IETP

#### A. FLEXURAL STRENGTH

By substituting the values of the flexural strengths (responses) from Table 3 into Eqns.(8) through (10), we obtain the coefficients (  $\beta_1, \beta_2 \dots \beta_{34}, \beta_{35} \dots \beta_{45}$ ) of the Scheffe's second degree polynomial for NFRC. Substituting the values of these coefficients into Eqn. (7) yield the polynomial model for the optimization of the flexural strength of NFRC (at both 14<sup>th</sup> day or 28<sup>th</sup> day) based on Scheffe's (5,2) lattice as stated under:

$$P^F = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5 \quad (22)$$

#### B. SPLIT TENSILE STRENGTH

By substituting the values of the split tensile strengths (responses) from Table 4 into Eqns.(8) through (10), we obtain the coefficients (  $\beta_1, \beta_2 \dots \beta_{34}, \beta_{35} \dots \beta_{45}$ ) of the Scheffe's second degree polynomial for NFRC. Substituting the values of these coefficients into Eqn. (7) yield the polynomial model for the optimization of the split tensile strength of NFRC (at both 14<sup>th</sup> day or 28<sup>th</sup> day) based on Scheffe's (5,2) lattice as given under:

$$P^S = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{15} X_1 X_5 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{25} X_2 X_5 + \beta_{34} X_3 X_4 + \beta_{35} X_3 X_5 + \beta_{45} X_4 X_5 \quad (23)$$

### 3.4. SCHEFFE'S (5,2) MODEL RESPONSES (FLEXURAL STRENGTH AND SPLIT TENSILE STRENGTH) FOR NFRC AT ECTP

#### A. FLEXURAL STRENGTH

By substituting the pseudo mix ratio of points C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub>, C<sub>4</sub>, C<sub>5</sub>, ... C<sub>45</sub> of Table 4 into Eqn.(22), we obtain the Scheffe's second degree model responses (flexural strength) for the control points of NFRC.

#### B. SPLIT TENSILE STRENGTH

By substituting the pseudo mix ratio of points C<sub>1</sub>, C<sub>2</sub>, C<sub>3</sub>, C<sub>4</sub>, C<sub>5</sub>, ... C<sub>45</sub> of Table 4 into Eqn.(23), we obtain the second degree model responses (split tensile strength) for the control points of NFRC.

### 3.5. TEST OF ADEQUACY OF NFRC MODEL RESULTS (FOR FLEXURAL STRENGTH AND SPLIT TENSILE STRENGTH) USING STUDENT'S – T -TEST

In this session, the test of adequacy is performed to determine how correlated the flexural and split tensile strengths results (lab responses) given in Tables 4 and model responses from the control points based on session 3.4 are. Using the Student's – T – test as the means of validation, the result shows that there are no significant differences between the experimental results and model responses. The steps involved in using the Student's – T - test have been explained by Nwachukwu and others (2022 c). Thus, the models are adequate for predicting the flexural and split tensile strengths of NFRC based on Scheffe's (5,2) simplex lattice.

### 3.6. RESULTS DISCUSSION

The highest obtainable flexural strengths of NFRC based on Scheffe's (5,2) lattice are **7.44 MPa** and **6.00 MPa** respectively for 28<sup>th</sup> and 14<sup>th</sup> day results. Similarly the maximum split tensile strengths of NFRC based on Scheffe's (5,2) lattice are **5.25 MPa** and **3.98 MPa** respectively for 28<sup>th</sup> and 14<sup>th</sup> day results. The corresponding optimum mix ratio is **0.70: 1: 1.00:1.80: 1.20** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Nylon Fibre respectively. The minimum flexural strength and split tensile strength are **4.08 MPa**, **3.80 MPa**, **3.08 MPa** and **2.50 MPa** respectively for the 28<sup>th</sup> day and 14<sup>th</sup> day results. The minimum values correspond to the mix ratio of **0.62:1:1.65:1.90: 0.65** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Nylon Fibre respectively. Thus, the Scheffe's model can be used to determine the NFRC flexural and split tensile strength of all points (1 - 45) in the simplex based on Scheffe's Second Degree Model for five component mixture.

## 4. CONCLUSION

So far in this work, the Scheffe's Second Degree Polynomial (5,2) has been presented and used to formulate a model for predicting the flexural and split tensile strengths of NFRC. In the first instance, the Scheffe's model was used to predict the mix ratio for evaluating both the flexural and split tensile strengths of NFRC. Through the use of Scheffe's (5,2) simplex model, the values of both strengths were determined at all 15 points (1 - 45). The result of the student's t-test shows that the strengths predicted by the models and the corresponding experimentally observed results are closely correlated. The optimum attainable strengths predicted by the model based on Scheffe's (5,2) model are as stated in the results discussion session. This is to say that with the Scheffe's (5,2) model, any desired strength of NFRC, given any mix ratio can be easily predicted and evaluated and vice versa. Thus, the application of this Scheffe's optimization model has reduced the problem of having to go through vigorous, time-consuming and laborious empirical mixture design procedures in order to obtain the desired design strengths of NFRC mixture. Finally, the use of Scheffe's optimization model has made partial replacement for any concrete component especially the conventional expensive steel reinforcement with inexpensive fibres possible.

## 5. REFERENCE

- [1] ACI Committee 211. (1994): "Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete (ACI 211.1R-91)"; American Concrete Institute, ACI Manual of Concrete Practice, Detroit, Michigan, USA.
- [2] ACI Committee 544. (1982): "State-of-the-Report on Fibre Reinforced Concrete, (ACI 544.1R-82)"; *Concrete International: Design and Construction*. Vol. 4, No. 5: Pp. 9-30, American Concrete Institute, Detroit, Michigan, USA.
- [3] ACI Committee 544. (1989): "Measurement of Properties of Fibre Reinforced Concrete, (ACI 544.2R-89)"; American Concrete Institute, Detroit, Michigan, USA.
- [4] Aggarwal, M.L. (2002): "Mixture Experiments: Design Workshop Lecture Notes", University of Delhi, India.
- [5] Ali, A., Aijaz, A., and Arsalan, M. (2018): A Study On Nylon Fibre Reinforced Concrete By Partial Replacement Of Cement With Metakaolin : A Literature Review; *International Research Journal of Engineering and Technology (IRJET)*, Vol.5, No. 3.
- [6] ASTM C33/C33M-18(2018); Standard Specification for Concrete Aggregates. ASTM International: West Conshohocken, PA, USA.
- [7] ASTM C78/C78M-22(2022); Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading). ASTM International: West Conshohocken, PA, USA.
- [8] ASTM C496/C496M-17 (2017); Standard Test Method for Splitting Tensile Strength of Cylindrical Concrete Specimens. ASTM International: West Conshohocken, PA, USA.
- [9] ASTM C1602/C1602M-22(2022); Standard Specification for Mixing Water Used in the Production of Hydraulic Cement Concrete. ASTM International: West Conshohocken, PA, USA.

- [10] British/European code (BS EN 12390-6) 2009. Testing concrete - Split tensile strength of test specimens. BSI - London.
- [11] British Standards Institution, BS 12 (1978): Ordinary and Rapid – Hardening Portland cement; London.
- [12] British Standards Institution, BS 1881-Part 118 (1983). Methods of Determination of Flexural Strength of Concrete Cube, London
- [13] Department of Environment, DOE(1988): Design of Normal Concrete Mixes , HMSO, London.
- [14] Egamana, S and Sule, S.(2017): Optimization of compressive strength of periwinkle shell aggregate concrete, Nigerian Journal of Technology (NIJOTECH), Vol. 36, No. 1, Pp.32- 38.
- [15] Ezech, J.C.& Ibearugbulam, O.M. (2009):“Application of Scheffe’s Model in Optimization of Compressive Cube Strength of River Stone Aggregate Concrete”;*International Journal of Natural and Applied Sciences*; Vol. 5, No. 4, Pp 303 – 308 .
- [16] Ezech, J.C., Ibearugbulam, O.M.& Anyaogu, L. (2010a):“Optimization of Compressive Strength of Cement-Sawdust Ash Sandcrete Block using Scheffe’s Mathematical Model”; *Journal of Applied Engineering Research*. Vol.4, No.4, Pp 487–494.
- [17] Ezech, J.C., Ibearugbulam, O.M.& Anya, U. C (2010b):“Optimization of aggregate composition of laterite/sand hollow Block Using Scheffe’s Simplex Method”;*International Journal of Engineering*., Vol.4, No.4, Pp 471 – 478.
- [18] Ganesh Kumar, R., Prasath, K.S. and Sudharsan, R.(2019): A Study on Waste Nylon Fiber in Concrete; International Journal of Research and Innovation in Applied Science (IJRIAS), Vol.4, No.2, Pp 125 – 126.
- [19] Hasan, M., Saidi, T., Jamil, M., Amalia, Z. and Mubarak, A.(2022): Mechanical Properties and Absorption of High-Strength Fiber-Reinforced Concrete (HSFRC) with Sustainable Natural Fibers; Innovative Concrete Composite: Materials and Structures with Sustainability-Centred Design, Buildings , Vol.12, No.12, available at <https://doi.org/10.3390/buildings12122262>
- [20] Hossain, M.A., Rahman, M.M., Morshed, A.Z. and Haque, S.M.(2012):— Investigation of the Effect of Nylon Fiber in Concrete Rehabilitation; Proceedings of the 1st International Conference on Civil Engineering for Sustainable Development, Khulna, Bangladesh
- [21] Hughes, B.P. (1971): The Economic Utilization of Concrete Materials, Proceedings of the Symposium on Advances in Concrete , Concrete Society, London
- [22] Ibearugbulam, O.M. (2006):“Mathematical Model for Optimization of Compressive Strength of Periwinkle Shell-Granite Aggregate Concrete”;*M.Eng.. Thesis*, Federal University of Technology, Owerri, Nigeria.
- [23] Jackson, N. and Dhir, R.K. (1996): Civil Engineering Materials, 5<sup>th</sup> Edition, Palgrave, New York.
- [24] Manikandan, K., Arun kumar, A., Deepak kumar, M., Manikandan, V. and Sathish kumar, K.(2017): Experimental Investigation On Nylon Fiber Reinforced Concrete; International Research Journal of Engineering and Technology (IRJET), Vol.4, No.3.
- [25] Mbadike, E. M. and Osadebe, N.N.(2013): Application Of Scheffe’s Model In Optimization Of Compressive Strength Of Laterite Concrete; Journal of Civil Engineering And Construction Technology, Vol. 4, No. 9, Pp. 265- 274.
- [26] Neville, A.M. (1990): Properties of Concrete; 3rd edition, Pitman, England.
- [27] Nwachukwu, K.C., Okafor, M., Thomas, B., Oputa, A.A., Okodugha, D.A. and Osaigbovo, M.E. (2017): “An Improved Model For The Optimization Of The Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC) Using Scheffe’s Second Degree Polynomials”; Research Journal of Civil Engineering, Vol. 3, No. 2
- [28] Nwachukwu, K. C., Njoku, K.O., Okorie, P. O., Akosubo, I.S., Uzoukwu, C. S., Ihemegbulem, E.O. and Igbojiaku, A.U (2022a):” Comparison Between Scheffe’s Second Degree (5,2) And Third Degree (5,3) Polynomial Models In The Optimization Of Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC)”; American Journal of Computing and Engineering (AJCE), Vol. 5, No. 1, Pp 1– 23.
- [29] Nwachukwu, K.C., Okodugha, D.A., Uzoukwu, C. S., Igbojiaku, A.U. and Okafor, M. (2022b): “Application Of Scheffe’s Second Degree Mathematical Model For The Optimization Of Compressive Strength Of Steel Fibre Reinforced Concrete (SFRC)”; International Journal of Advances in Engineering and Management (IJAEM) , Vol. 4, No. 3.
- [30] Nwachukwu, K.C., Okorie, P.O., Ihemegbulem, E.O., Kemebiye Dimaro, Uzoukwu, C. S., and Maduagwu, J.C.. (2022c): “The Use Of Scheffe’s Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC)”; International Journal of Innovative Science and Research Technology (IJISRT), Vol. 7, No. 1



- [31] Nwachukwu, K.C., Ozioko, H.O., Okorie, P.O, and Uzoukwu, C. S. (2022d): "Application Of Scheffe's Mathematical Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRFC)"; International Journal of Advances in Engineering and Management (IJAEM) , Vol. 4, No. 2, Pp 850-862
- [32] Nwachukwu, K. C., Okodugha, D.A., Igbojiaku, A.U., Attah, U.G. And Ihemegbulem, E.O. (2022e): Application Of Scheffe's Third Degree Regression Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC); International Journal Of Advance Research And Innovative Ideas In Education (IJARIIE) , Vol. 8, No. 2.
- [33] Nwachukwu, K. C., Okodugha, D.A., Edike, O., Okafor, M.. And Attah, U.G. (2022f): Application Of Modified Scheffe's Third Degree Polynomial Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRFC); International Journal of Trend In Scientific Research And Development (IJTSRD) , Vol. 6, No. 3.
- [34] Nwachukwu, K.C., Okodugha, D.A., Akosubo, I.S. and Atulomah, F.K (2022g): Optimization of Compressive Strength of Steel Fibre Reinforced Concrete (SFRC) Using Scheffe's Third-Degree Regression Model; Iconic Research And Engineering Journals (IRE), Vol. 5, No. 11.
- [35] Nwachukwu, K.C., Onwuegbuchulem, U.C., Atulomah, F.K. and Ihemegbulem, E.O. (2022h): Optimization of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC) ; Iconic Research And Engineering Journals (IRE), Vol. 5, No. 12
- [36] Nwachukwu, K.C., Thomas, B., Oguaghamba, O., Onwuegbuchulem, U.C., Igbojiaku, A.U., Atulomah, F.K and Okafor, M.. (2022i): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) Using Scheffe's Second Degree Model; international Journal of Engineering Inventions (IJEI), Vol.11 , No.8, Pp.40-50.
- [37] Nwachukwu, K.C., Oguaghamba, O., Akosubo, I.S., Atulomah, F.K. and Igbojiaku, A.U. (2022j): Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRFC); International Journal Of Advanced Research And Innovative Ideas In Education (IJARIIE), Vol. 8, No. 5. Pp. 376 -392.
- [38] Nwachukwu, K.C., Edike, O., Oguaghamba, O., Onwuegbuchulem, U.C and Egbulonu, B.A. (2022k): The Application Of Scheffe's Model For The Optimization Of Mussel Shells Fibre Reinforced Concrete (MSFRC); International Journal of Engineering Inventions (IJEI), Vol. 11, No. 9. Pp. 47- 57
- [39] Nwachukwu, K.C., Ozioko, H.O., Akosubo, I.S., Oguaghamba, O., Okafor, M., Onyechere, I.C. and Ikpa, P.N (2022l): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model; Paper presented at the Proceedings of the 1<sup>ST</sup> International Conference on Advances in cement and concrete research (ICACCR-2022) Organized by Ladoke Akintola University of Technology (LAUTECH), Nigeria. Theme: Cement and Concrete Innovative Materials For Sustainable Development.
- [40] Nwachukwu, K.C and Opara, H.E. (2023): Maximizing And Exploring The Benefits Of Using Latest Innovative Materials (LIM) And Low Cost Automation (LCA) For The Rapid Economic Growth Of The Cement Manufacturing Industry; Paper presented at the Conference Proceedings of the Nigeria Society of Engineers (Abuja 2023), Nov. 27 – Dec.1, 2023.
- [41] Nwachukwu, K.C., Ozioko, H. O., Mama, B.O., Oguaghamba, O., and Atulomah, F.K. (2023a): The Application of Scheffe's Third Degree Regression Model In The Optimization Of Compressive Strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC); International Journal of Research and Analytical Reviews (IJRAR), Vol. , No.. Pp.
- [42] Nwachukwu, K.C., Oguaghamba, O., Ozioko, H. O. and Mama, B.O. (2023b): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) Using Scheffe's (6,3) Model; International Journal of Trend In Scientific Research and Development (IJTSRD), Vol.7 , No.2, Pp. 737-754
- [43] Nwachukwu, K. C., Mathew, C.C., Mama, B.O., Oguaghamba, O. And Uzoukwu, C.S. (2023c): Optimization Of Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC) : International Journal Of Advance Research And Innovative Ideas In Education (IJARIIE) , Vol. , No.
- [44] Nwachukwu, K. C., Oguaghamba, O., Akosubo, I.S., Egbulonu, B.A., Okafor, M. and Mathew, C.C. (2023d): The Use Of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre

- Reinforced Concrete (AFRC); Research Inventory: International Journal of Engineering and Science (IJES) , Vol. , No. .
- [45] Nwachukwu, K.C., Nnegha, A.R., Senator, W.U., Okere, C.P. , Nmecha, T.M., Obi, C.G., Etukudoh, E.E. and Ezeike, U.C.(2024a): Determination Of Optimized Flexural And Split Tensile Strengths Of PSA-MSA-Cement Concrete (PMCC) Using Scheffe's (6, 2) Model; International Journal of Research Publication and Reviews [IJRPR], Vol. 5, No. 5, Pp.
- [46] Nwachukwu, K. C., Anyachi, C.M., Njoku, I.C., Olinya, I.A., Nkama, J., Egbara, T.E., Anike, V.A., Nwabueze, C.S and Ejiofor, H.C.(2024b): Investigation Of Compressive Strength Property Of PSA - SSA-Cement Concrete (PSCC) Using Scheffe's (6, 2) Model; International Journal of Research Publication and Reviews [IJRPR], Vol. 5, No. 6, Pp.
- [47] Nwachukwu, K. C., Edike, O., Mathew, C.C., Mama, B.O. and Oguaghamba, O..(2024c): Evaluation Of Compressive Strength Property Of Plastic Fibre Reinforced Concrete (PLFRC) Based On Scheffe's Model; International Journal of Research Publication and Reviews [IJRPR], Vol. 5, No. 6, Pp.
- [48] Nwachukwu, K. C., Edike, O., Mathew, C.C., Oguaghamba, O. and Mama, B.O.(2024d): Investigation of Compressive Strength Property of Hybrid Polypropylene - Nylon Fibre Reinforced Concrete (HPNFRC) Based on Scheffe's (6,3) Model; International Journal of Research Publication and Reviews [IJRPR], Vol. 5, No. 6, Pp. 4020- 4039
- [49] Nwachukwu, K. C., Nwachukwu, A.N., Njoku, C.F. , Uzoukwu, C.S., Ozioko, H.O. and Mathew, C.C.,(2024e): Application Of Scheffe's (5,3) Model In The Compressive Strength Determination Of Mussel Shells Fibre Reinforced Concrete (MSFRC); Goya Journal, Vol. 17, No. 6.
- [50] Nwakonobi, T.U and Osadebe, N.N (2008):"Optimization Model for Mix Proportioning of Clay- Ricehusk-Cement Mixture for Animal Buildings"; *Agricultural Engineering International:the CIGR Ejournal,Manuscript BC 08 007*, Vol x
- [51] Obam, S.O. (2006). The Accuracy of Scheffe's Third Degree over Second Degree Optimization Regression Polynomials, Nigerian Journal of Technology, Vol. 2, No.25, Pp 1 – 10.
- [52] Obam, S.O.(2009):"A Mathematical model for Optimization of Strength of concrete : A case study for shear modulus of Rice Husk Ash Concrete." *Journal of Industrial Engineering International I*; Vol.5, No.9,Pp 76 – 84.
- [53] Okere, C.E., (2006):"Mathematical Models for Optimization of the Modulus of rupture of concrete"; M.Eng. Thesis, Civil Engineering Department, Federal University of Technology, Owerri.
- [54] Onwuka,D.O, Okere, C.E.,Arimanwa, J.I. and Onwuka, S.U.(2011):"Prediction of Concrete Mix ratios using Modified Regression Theory, *Computer Methods in Civil Engineering*, Vol. 2. No.1 Pp.95- 107.
- [55] Oyenuga, V.O. (2008): Simplified Reinforced Concrete Design, 2<sup>nd</sup> Edition, Asros Ltd, Lagos, Nigeria.
- [56] Samrose, S., and Mutsuddy, R.(2019):—Durability Of Nylon Fiber Reinforcement Concretel; Proceedings on International Conference on Disaster Risk Management, .Dhaka, Bangladesh.
- [57] Scheffe, H. (1958):Experiment with Mixtures"; *International Journal of Royal Statistics Society*, Series B, Vol.20, Pp. 344-360.
- [58] Shacklock, B.W. (1974): Concrete Constituents And Mix Proportions, The Cement and Concrete Association, London.
- [59] Shetty, M.S. (2006): Concrete Technology (Theory and Practice), S. Chand and Company Ltd, India.
- [60] Song, P.O., Hwang, S. and Sheu, B.C.(2005): Strength Properties of Nylon and Polypropylene-fiber-Reinforced Concretel; Science Direct;Vol. 35, No.8,Pp 1546 – 1550, India.
- [61] Syal, I.C. and Goel, A.K. (2007): Reinforced Concrete Structures, S.Chand , India.