VARIATIONAL ITERATIVE DIFFERENTIAL TRANSFORM METHOD FOR SOLVING VIRAL ILLNESS WITH INFECTION DEFENSE

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**Abstract:**

 The extension of the viral illness with infection defense has been modelled by the SIR epidemic model. The model is of the form a system of linear differential equations where exact solution is difficult to determind. In this paper, the model is solved by variational iterative differential transform method. In this case of computation is divided into a finite number of subintervals. At each subinterval we apply continuity condition to ensure the continuity solutios. VIM uses the Lagrange multiplier to construct the correction functional for the problem while DTM uses the transformed function of the original nonlinear system.

Keywords: SIR epidemic model, Variational Iterative Method, Fractional Differential transform method.

**Introduction:**

Epidemiology studies the spread of diseases in population and primarily the human population. The S.I.R model was introduced by J.D. Murray and has played a major role in mathematical epidemiology. In the model, a population is divided into three groups: the Susceptible(S), the Infectious(I), and the Recovered(R). The total population (N) is

N = S + I + R

The susceptible are those who are not infected and not immune, the infective are those who are infected and can transmit the disease, and the recovered are those who have been infected, have recovered and are permanently immune. Infectious diseases are health issues characterized by a departure from sound state of health triggered by structural changes such that the normal body fuction is impaired, making it extremely difficult for the organs of the body to perform properly. Infectious diseases due to their ability to spread from one species to another through replicating agent. The spread of an infectious disease may be attributed to one or more different pathways such as contact with infected indiduals. The infectious agents may be ingested through body.

 Since the incubation period for venereal diseases is usually quite short in gonorhea, for example, it is three to seven days- when compared to the infectious period, we uses an extension of the simple epidemic model. We divide the promiscuous male population into susceptibles,S, Infectious, I, and a removed class, R; the similar female groups we denote by S\*, I\*, R\*. If we do not include any transition from the removed class to the susceptible group, the infection dynamics is schematically. The most important aspects of defense against infectious diseases is unquestionably surveillance which characterises the pattern of each disease. Although ther are social problems associated with gathering data on the number of people who the HIV, it is unlikely that the will be contained if this information is not made available.

 The past decade has been characterized by the fast emergence and development of a considerable theory of epidemics. The much celebrated threshold dynamics which one the major quantity in epidemiology was derived in 1927 by Kermack and Makendrick. The historic work of Kermack and Makendrick was followed by the radical work of Bartlett who investigated models and data to examine the circumstances that influence disease persistence in host popultions. Perhaps, the first groundbreaking book on mathematical modelling regarding the epidemiological systems was introduced by Bailey which led to the appreciation of the significance of modelling in decision making regarding the public health issues. Given the range of epidemic diseases studied since 1950s, a remarkable class of epidemic models has been designed.

**Preliminaries:**

 A compartmental model is adopted to analyze the transmission dynamics of disease in a human population. The model is divided into subpopulations based on the epidemiological status of individuals in the population. The susceptible population is generated from the daily recruitment of birth at the rate. It is increased as a result of loss of immunity after recovery at the rate and decreases due to vaccination and natural death at the rate, reapectively. The infectious class is generated at the rate when there is interaction between the susceptible and the infectious individuals. The infectious class however reduces through the natural death rate and through the recovery from infection and the disease induced death at the rates and respectively. Furthermore, the Recovery subclass is generated from vaccinated susceptible subpopulation and recovered infected individuals at the rate and respectively. To indicate mathematically, we have:

**Definition of the Reimann Liouville Fractional Integral:**

The Reimann-Liouville definition. For αϵ [n-1, n), the α- derivative of f is

 **Definition of Caputo Derivative:**

For α ∈ [n−1, n), the α derivative of f is

**Some Important Method** .

1. **Grunwald-Letnikov method**

Another very obvious approach for the discretization of Fractional differential and integral operators is based on straightforward generalization of concepts from classical calculus to the fractional case .specifically, it is well known that an integer order derivative can be written as a differential quotient viz.

1. **Fractional differential transform method:**

There are several approaches to the generalization of the notion of differentiation to fractional orders. The fractional differentiation in Riemann–Liouville sense is defined by

For let us expand the analytical and continuous function f(x) in terms of a fractional power series as follows

 Where is the order of fraction and F(k) is the fractional differential transform of f(x). concerning the practical applications encounterd in various brances of science, the fractional initial conditions are frequently not available, and it may not be clear what their physical meaning is. Therefore, the definition in above equation should be modified to deal with integer ordered initial conditions in Coputo sense as follows:

Since the initial conditions are implemented to the integer order derivatives, the transformation of the initial conditions are defined as follows:

, for

where, n is the order of FDE considered.

1. **Variational Iteration Method:**

 According to the variational iteration method of we consider the general differential equation

**Lp+Np = q(x)**

Where, L is a linear operator, N is a nonlinear operator and d(x) is an homogeneous term. We can construct a correctional function as follows

**pn+1 = pn(x)+**

where ε is a Lagrangian multiplier which can identified optimally via variational theory. The subscript n denotes the nth approximation an pn’ is considered as a restricted variation. Consider the stationary condition of the above correction functional, then the Lagrange multiplier can be expressed as

**ε1(w) =**

where m is the highest order of the differential equation.

**Problem Discription:**
We consider the SIR model given by the system of ordinary differential equations [10]

Here, represents the susceptible group. denotes the infected group. is the removed group including the vaccinated and recovered people with permanent immunity. is the total number of population and is the time variable.

Some assumptions for this model are as follows [10]. The vaccination is effective. Parameter represents the proportion of people vaccinated at birth each year, where ; the rest having the proportion is susceptible. Parameter represents the birth rate. Parameter is the average contact rate between a susceptible individual and an infected individual, where a susceptible individual will move into the infected group when the susceptible individual have contact with an infected individual. Prameter is the recovery rate of an infected individual to enter into the removed group. The natural death rate is denoted by , where it is unequal to the birth rate .

Based on our assumptions above, all parameters , are nonnegative constants. Here, the total population varies with respect to time , as we have assumed that . To verify that the total population is not constant, we can add Eqs. (1)-(3) to obtain

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Scaling each group by the total population, we obtain new variables . Using these new variables, we normalise each group, which means . Knowing Eq. (4) and substituting into Eqs. (1)(3), we obtain the normalised system for the SIR model,

In which parameter is eliminated due to algebraic operations, so parameter does not influence the dynamics of the normalised system. Qualitative analysis of the SIR model (1)-(3) can be carried out by analysing normalised system (5)-(7).

**Theorem 1.**

Let be continuous in the given domain containing , then any solution to the initial value problem is also a solution to the integral equation

and conversely.

**Proof.**
 The proof of this theorem is straightforward and available in the work of Agarwal and O’Regan. Based on this theorem, to solve intial value problem, Picard’s successive approximations take the intial approximation.

and the next approximations

where . We shall use this idea of Picard's successive approximations to construct the successive approximation method for Eqs. (5)-(7)

Considering (5)-(7) with given initial conditions , the successive approximation method takes the initial approximations

and the next approximations

where .

**SIR model for Viral Illness:**
The SIR model which was introduced by W. O. Kermack has played an important role in mathematical epidemiology. In the model, a population is divided into three groups:

* Susceptible individuals : Represent individuals that were not contaminated by Covid 19 at time
* Infected individuals : Represent individuals infected by desease at time
* Recovered individuals : Represent individuals that have recovered and death from Covid 19 at time .

There are some important assumptions for the simple SIR Model. The outbreak is short-lived. is the total size of the population. The total population is large and closed, and its size remains. No natural birth or natural death occurs. Hence, the population is constant over time. For any time and . The number of individuals in each compartment must be integer, but if the population size is sufficiently large, it is possible to treat and as continuous variables. Initially, in the absence of infection we have and , i.e., and we assume that the whole population is susceptible. : The time dependent rate of transmission of the disease from susceptible, : The rate of recovery from infected to recovered.We assume that the carriers leave the class infected (I) at a constant rate and enter directly to the class recovered . The motion line of the model is as follows.

 (S) (I) (R)

SIR model considered that everyone in the population has an equal probability of getting infected. The SIR model can be mathematically represented as follows.

This mathematical model works on a few assumptions. SIR model considered that everyone in the population has equal probability of getting infected model reliability depends on the quality of data, it is assumed that the available on the open platform is correct.

**Conclusion:**

Studies on the modelling of epidemic diseases have gained considerable importance and popularity. Varitional iteration methods can approximate exact solutions at any given time and the approximate solutions are continuous at all time. The SIR model of differential equations can be solved conveniently using a numerical solver for differential equations. In this paper, fractional differential transform method and variational iterative method has been successfully employed to obtain approximate solution of SIR model with initial condition. These two methods shows that both are in excellent agreement which indicates their effectiveness and reliability. It provides more realistic series solutions that converge very rapidly with exact solutions.

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