**Gβ\*- CONTINUOUS MAPS IN GRILL TOPOLOGICAL SPACES**

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**ABSTRACT**

 In this paper, we have introduced and analyze a new class of Gβ\*-continuous map in grill topological space. Its relation to various other continuous functions are investigated.

**Keywords:** Gβ\*- closed sets, Gβ\*- continuous maps,

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**I. INTRODUCTION**

 Choquet introduced the concept of grills on a topological space [3]. Hatir and Jafari [6] obtained a new decomposition of continuity in terms of grills. Antony Rex Rodgio and Jessie Theodore [2] introduced $β^{⋆}$- continuous maps in topological space using $β^{⋆}$- closed sets. In this paper, we have introduced and analyze a new class of Gβ\*-continuous map in grill topological space (X, $τ$, G).

**II. PRELIMINARIES**

**2.1 Definition:** [3] A non-null collection G of subsets of a topological spaces *X* is said to be a grill on Xif

1. $ϕ\notin G$
2. A ∈ G and A ⊆ B $⟹$B ∈ G,
3. A, B ⊆ X and A ∪ B ∈ G $⟹$ A ∈ G or B ∈ G.

**2.2 Definition:** Let (X, τ, G) be a grill topological space. A subset A in X is said to be

1. ϕ -open [6] if A ⊆ Int(ϕ(A)),
2. Gα-open[1] if A ⊆ Int(Ψ(Int(A))),
3. G-preopen [6] if A ⊆ Int(Ψ(A)),
4. G-semi-open [1] if A ⊆ Ψ(Int(A)),
5. G$β$ –open [1] if A ⊆ Cl(Int(Ψ(A))).

The family of all Gα-open (resp. G-preopen, G-semi-open, G$β$ -open) sets in a grill topological space (X, τ, G) is denoted by GαO(X) (resp. GPO(X), GSO(X), GβO(X)).

**2.3 Definition:[12]** A subset A of X is called a Gβ\*-closed set if G$βcl(A)⊆int(U)$ whenever A$⊆$ U and U is ω-open in X.The class of all Gβ\*-closed sets in X is denoted by Gβ\*C(τ). That is Gβ\*C(τ)={A$⊂$ X: A is Gβ\*-closed in X}.

**2.4 Definition:** A function f : (X, τ, GX) → (Y, σ, GY) is said to be

1. grill α-continuous[1] if the inverse image of each open set of Y is Gα-open.
2. grill semi –continuous[1] if the inverse image of each open set of Y is G-semi-open
3. grill pre - continuous [6] if the inverse image of each open set of Y is G-preopen.
4. grill $β$ –continuous [1] if the inverse image of each closed set of Y is G$β$ –closed.
5. grill g- continuous [7] if the inverse image of each closed set of Y is Gg-closed.
6. grill rg- continuous [10]if the inverse image of each closed set of Y is Grg-closed.
7. grill gsp –continuous[12] if the inverse image of each closed set of Y is Ggsp-closed.
8. grill ω – continuous [12] if the inverse image of each closed set of Y is Gω-closed.
9. grill \*g – continuous[12] if the inverse image of each closed set of Y is G \*g-closed
10. grill g\* - continuous[11] if the inverse image of each closed set of Y is Gg\*-closed.
11. grill sg –continuous[9] if the inverse image of each closed set of Y is Gsg-closed.
12. grill gs-continuous [13]if the inverse image of each closed set of Y is Ggs-closed
13. grill pre-semi-continuous[12] if the inverse image of each closed set of Y is G-pre-semi closed
14. grill $\hat{η}^{\*}$-continuous[12] if the inverse image of each closed set of Y is G$\hat{η}^{\*}$-closed

# III. Gβ\*- CONTINUOUS MAPS IN GRILL TOPOLOGICAL SPACES

**3.1 Definition**: A map f: X$\rightarrow $Y is called Gβ\*- continuous if $f^{-1}$(V) is Gβ\*- closed in X for every closed set V in Y.

**3.2 Proposition:** Every continuous (respectively Gα-continuous, G-semi continuous) map is Gβ\*- continuous but not conversely.

**Proof:** The proof follows from the fact that every closed (respectively Gα- closed, G- semi closed) set is Gβ\*-closed.

The converses of Proposition is shown by the following example

**3.3 Example:** Let X = {a, b, c}, Y = {p, q}, τ = {$ϕ$, {a}, X}, $σ$ = {$ ϕ$,{q},Y}, $G\_{X}$ = {{a},{a, b}, {a, c}, X} and $G\_{Y}$ = {{q}, Y}. The map f: X$\rightarrow $Y define by f(a) = f(b) = p and f(c)=q is Gβ\* continuous. However f is none of G-semi-continuous, Gα-continuous and continuous since for the closed set U= {p} in Y, $f^{-1}$(U) = {a, b} is none of G-semi closed, Gα-closed and closed in (X, τ, $G\_{X}$) but it is Gβ\* closed in (X, τ, $G\_{X}$).

**3.4 Proposition**: Every Gβ\*-continuous map is Ggsp-continuous (resp. G$\hat{η}^{\*}$- continuous) but not conversely.

**Proof:** Since every Gβ\*-closed set is Ggsp-closed (resp.G$\hat{η}^{\*}$-closed)

**3.5 Example:** Let X={a, b, c, d}, Y={p, q, r}, τ={$ ϕ$,{a, b}, X}, σ= {$ ϕ$,{p},{r},{p, r},Y}, $G\_{X}$ = {{a},{a, b}, {a, c},{a, d}, {a, b, c},{a, b, d},{a, c, d}, X} and $G\_{Y}$ = {{q},{p,q},{q, r}, Y}. Define f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) by f(a)=f(d)=p, f(b)=q, f(c)=r. Then it can be seen that f is Ggsp-continuous and G$\hat{η}^{\*}$-continuous, but not Gβ\*continuous, because for the closed set {q} in Y, $f^{-1}$({q}) = {b} is not Gβ\*- closed in (X, τ,$ G\_{X}$).

**3.6 Proposition**: The following examples show that Gβ\*-continuity is independent of Gω-continuity and Grg-continuity.

**3.7 Example**: Let X=Y= {a, b, c}, τ= {$ ϕ$, {a},{b},{a, b}, X}, σ={$ ϕ$,{a, c},Y}, $G\_{X}$ = {{b}, {a, b}, {b, c}, X} and $G\_{Y}$ = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map. Here Gβ\*c(τ)=P(X)-{a, b}. Then f is Gβ\*-continuous but none of Gω- continuity and Grg-continuous, since {b} is closed in Y but $f^{-1}$({b}) = {b} is none of Gω- closed and Grg-closed in (X, τ,$ G\_{X}$).

**3.8 Example:** Let X = Y = {a, b, c}, τ = {$ ϕ$,{a},{b, c},X}, σ = {$ ϕ$,{a, c},Y},

$G\_{X}$ = {{b},{a, b}, {b, c}, X} and $G\_{Y}$ = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map. Here Gβ\*c(τ)= {$ϕ$, {a},{b, c}, X}. Then f is Gω-continuous and Grg continuous but not Gβ\*-continuous for $f^{-1}$({b})={b} is not Gβ\*-closed in (X, τ,$ G\_{X}$).

**3.9 Proposition**: The following examples, show that Gβ\*-continuity is independent of Gg-continuity, Gsg-continuity and Ggs-continuity.

**3.10 Example:** Let X= {a, b, c}=Y , τ={$ ϕ$,{a},{b, c},X}, σ={$ ϕ$,{b}, Y}, $G\_{X}$ = {{b},{a, b}, {b, c}, X} and $G\_{Y}$ = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map. Then f is Gg-continuous, Ggs-continuous and Gsg-continuous, but not Gβ\*-continuous because for the closed set {a, c}, $f^{-1}$ ({a, c}) = {a, c} is not Gβ\*-closed in (X, τ,$ G\_{X}$).

**3.11 Example**: Let X=Y= {a, b, c}, τ= {$ ϕ$, {a}, {b}, {a, b}, X}, σ={$ ϕ$,{a, c},Y}, $G\_{X}$ = {{b},{a, b}, {b, c}, X} and $G\_{Y}$ = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map. Then f is Gβ\*-continuous but none of Gg- continuous, Gsg-continuous and Ggs-continuous because for the closed set {b} in Y, $f^{-1}$({b}= {b}is Gβ\*-closed but none of Gg-closed, Gsg-closed and Ggs-closed.

**3.12 Proposition**: The following examples show that Gβ\*-continuity is independent of G-pre-continuity, G-semi-pre-continuity and G-pre-semi-continuity.

**3.13 Example**: Let X=Y= {a, b, c, d}, τ= {$ ϕ$, {a, b}, X}, σ= {$ ϕ$, {c}, Y}, $G\_{X}$ = {{b},{a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X} and $G\_{Y}$ = {{a},{a, b},{a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}, Y}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map. Then f is Gβ\*-continuous but none of G-pre-continuous, G-semi-pre-continuous and G-pre-semi-continuous, since {a, b, d} is closed in Y but $f^{-1}$({a, b, d}) = {a, b, d} is none of G-preclosed, G-semi-preclosed and G-pre-semiclosed in (X, τ,$ G\_{X}$).

**3.14 Example:** Let X= {a, b, c}=Y , τ={$ ϕ$,{a},{b, c},X}, σ={$ ϕ$,{c}, Y}, $G\_{X}$ = {{b},{a, b}, {b, c}, X} and $G\_{Y}$ = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map. Then f is G-pre-continuous, G-semi-pre-continuous and G-pre-semi-continuous, but not Gβ\*-continuous because for the closed set {a, b}, $f^{-1}$ ({a, b}) = {a, b} is not Gβ\*-closed in (X, τ,$ G\_{X}$).

**3.15 Proposition**: The following examples show that Gβ\*-continuity is independent of G\*g-continuity and Gg\*-continuity.

**3.16 Example**: Let X= {a, b, c}=Y, τ = {$ ϕ$, {a}, {b}, {a, b}, X}, σ= {$ ϕ$, {a, c}, Y}, $G\_{X}$ = {{b}, {a, b}, {b, c}, X} and $G\_{Y}$ = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map. Then f is Gβ\*-continuous but neither Gg\*- continuous nor G\*g-continuous because for the closed set {b} is Gβ\*-closed but neither Gg\*-closed nor G\*g-closed in (X, τ,$ G\_{X}$).

**3.17 Example**: Let X= {a, b, c, d}, Y= {a, b, c}, τ = {$ ϕ$, {a, b}, X}, σ= {$ ϕ$, {a, c}, Y}, $G\_{X}$={{a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d},X} and $G\_{Y}$ = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map. Then f is Gg\*- continuous and hence G\*g-continuous but not Gβ\*-continuous because for the closed set {b} is Gg\*-closed and hence G\*g-closed but not Gβ\*-closed in (X, τ,$ G\_{X}$).

From the above discussion are shown in the following implications:

Gβ\*- continuous

 G-semi continuous

 Gα-continuous

Ggsp-continuous

Grg-continuous

Gω-continuous

G$\hat{η}^{\*}$- continuous

Gsg-continuous

Continuous

Ggs-continuous

Gg-continuous

G semi pre-continuous

G pre-continuous

G pre semi-continuous

G\*g-continuous

Gg\*- continuous

**3.18 Proposition**: The composition of two Gβ\*-continuous maps need not be Gβ\*-continuous.

**3.19 Example**: Let X=Y={a, b, c}, Z={p, q}, τ={$ ϕ$,{a},{b},{a, b}, X}, σ={$ ϕ$, {a}, Y}, η={$ ϕ$,{p}, Z} $G\_{X}$ = {{b},{a, b}, {b, c}, X}, $G\_{Y}$ = {{a},{a, b}, {a, c}, Y} and $G\_{Z}$ = {{p}, Z}. Let f: (X, τ,$ G\_{X}$) $\rightarrow $(Y, σ, $G\_{Y}$) be the identity map and define a map g: (Y, σ,$ G\_{Y}$) $\rightarrow $(Z, η, $G\_{Z}$) by g(a)=g(b)=q and g(c)=p. Then both f and g are Gβ\*- continuous. Consider the closed set A= {q} in (Z, η). For this set $(g∘f)^{-1}$(A) = $f^{-1} (g^{-1}$(A)) = {a, b} is not Gβ\*-closed in (X, τ,$ G\_{X}$). Therefore $g∘f$ is not Gβ\*-continuous.

**3.20 Proposition**: If f: X$\rightarrow $Y is Gβ\*-continuous and g: Y$\rightarrow $Z is continuous, then their composition $g∘f$: X$\rightarrow $Z is Gβ\*-continuous.

**Proof:** Clearly follows from definitions.

**3.21 Theorem:** A map f: X$\rightarrow $Y is Gβ\*- continuous if and only if $f^{-1}$(U) is Gβ\*-open for every open set U in Y.

**Proof:** Let f: X$\rightarrow $Y be Gβ\*-continuous and U be an open set in Y. Then $f^{-1}$($U^{C}$) is Gβ\* closed in X. But $f^{-1}$($U^{C}$) = $(f^{-1}(U))^{C}$ and so $f^{-1}$(U) is Gβ\*-open in X. Converse is similar.

**V. CONCLUSION**

 In this paper, we have introduced and analyzed Gβ\*-continuous map in grill topological space. Its relation to various other continuous functions are investigated. Also verified the composition of two Gβ\*-continuous maps need not be Gβ\*-continuous.

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