**Gβ\*- CONTINUOUS MAPS IN GRILL TOPOLOGICAL SPACES**

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**ABSTRACT**

In this paper, we have introduced and analyze a new class of Gβ\*-continuous map in grill topological space. Its relation to various other continuous functions are investigated.

**Keywords:** Gβ\*- closed sets, Gβ\*- continuous maps,

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**I. INTRODUCTION**

Choquet introduced the concept of grills on a topological space [3]. Hatir and Jafari [6] obtained a new decomposition of continuity in terms of grills. Antony Rex Rodgio and Jessie Theodore [2] introduced - continuous maps in topological space using - closed sets. In this paper, we have introduced and analyze a new class of Gβ\*-continuous map in grill topological space (X, , G).

**II. PRELIMINARIES**

**2.1 Definition:** [3] A non-null collection G of subsets of a topological spaces *X* is said to be a grill on Xif

1. A ∈ G and A ⊆ B B ∈ G,
2. A, B ⊆ X and A ∪ B ∈ G A ∈ G or B ∈ G.

**2.2 Definition:** Let (X, τ, G) be a grill topological space. A subset A in X is said to be

1. ϕ -open [6] if A ⊆ Int(ϕ(A)),
2. Gα-open[1] if A ⊆ Int(Ψ(Int(A))),
3. G-preopen [6] if A ⊆ Int(Ψ(A)),
4. G-semi-open [1] if A ⊆ Ψ(Int(A)),
5. G –open [1] if A ⊆ Cl(Int(Ψ(A))).

The family of all Gα-open (resp. G-preopen, G-semi-open, G -open) sets in a grill topological space (X, τ, G) is denoted by GαO(X) (resp. GPO(X), GSO(X), GβO(X)).

**2.3 Definition:[12]** A subset A of X is called a Gβ\*-closed set if G whenever A U and U is ω-open in X.The class of all Gβ\*-closed sets in X is denoted by Gβ\*C(τ). That is Gβ\*C(τ)={A X: A is Gβ\*-closed in X}.

**2.4 Definition:** A function f : (X, τ, GX) → (Y, σ, GY) is said to be

1. grill α-continuous[1] if the inverse image of each open set of Y is Gα-open.
2. grill semi –continuous[1] if the inverse image of each open set of Y is G-semi-open
3. grill pre - continuous [6] if the inverse image of each open set of Y is G-preopen.
4. grill –continuous [1] if the inverse image of each closed set of Y is G –closed.
5. grill g- continuous [7] if the inverse image of each closed set of Y is Gg-closed.
6. grill rg- continuous [10]if the inverse image of each closed set of Y is Grg-closed.
7. grill gsp –continuous[12] if the inverse image of each closed set of Y is Ggsp-closed.
8. grill ω – continuous [12] if the inverse image of each closed set of Y is Gω-closed.
9. grill \*g – continuous[12] if the inverse image of each closed set of Y is G \*g-closed
10. grill g\* - continuous[11] if the inverse image of each closed set of Y is Gg\*-closed.
11. grill sg –continuous[9] if the inverse image of each closed set of Y is Gsg-closed.
12. grill gs-continuous [13]if the inverse image of each closed set of Y is Ggs-closed
13. grill pre-semi-continuous[12] if the inverse image of each closed set of Y is G-pre-semi closed
14. grill -continuous[12] if the inverse image of each closed set of Y is G-closed

# III. Gβ\*- CONTINUOUS MAPS IN GRILL TOPOLOGICAL SPACES

**3.1 Definition**: A map f: XY is called Gβ\*- continuous if (V) is Gβ\*- closed in X for every closed set V in Y.

**3.2 Proposition:** Every continuous (respectively Gα-continuous, G-semi continuous) map is Gβ\*- continuous but not conversely.

**Proof:** The proof follows from the fact that every closed (respectively Gα- closed, G- semi closed) set is Gβ\*-closed.

The converses of Proposition is shown by the following example

**3.3 Example:** Let X = {a, b, c}, Y = {p, q}, τ = {, {a}, X}, = {,{q},Y}, = {{a},{a, b}, {a, c}, X} and = {{q}, Y}. The map f: XY define by f(a) = f(b) = p and f(c)=q is Gβ\* continuous. However f is none of G-semi-continuous, Gα-continuous and continuous since for the closed set U= {p} in Y, (U) = {a, b} is none of G-semi closed, Gα-closed and closed in (X, τ, ) but it is Gβ\* closed in (X, τ, ).

**3.4 Proposition**: Every Gβ\*-continuous map is Ggsp-continuous (resp. G- continuous) but not conversely.

**Proof:** Since every Gβ\*-closed set is Ggsp-closed (resp.G-closed)

**3.5 Example:** Let X={a, b, c, d}, Y={p, q, r}, τ={,{a, b}, X}, σ= {,{p},{r},{p, r},Y}, = {{a},{a, b}, {a, c},{a, d}, {a, b, c},{a, b, d},{a, c, d}, X} and = {{q},{p,q},{q, r}, Y}. Define f: (X, τ,) (Y, σ, ) by f(a)=f(d)=p, f(b)=q, f(c)=r. Then it can be seen that f is Ggsp-continuous and G-continuous, but not Gβ\*continuous, because for the closed set {q} in Y, ({q}) = {b} is not Gβ\*- closed in (X, τ,).

**3.6 Proposition**: The following examples show that Gβ\*-continuity is independent of Gω-continuity and Grg-continuity.

**3.7 Example**: Let X=Y= {a, b, c}, τ= {, {a},{b},{a, b}, X}, σ={,{a, c},Y}, = {{b}, {a, b}, {b, c}, X} and = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,) (Y, σ, ) be the identity map. Here Gβ\*c(τ)=P(X)-{a, b}. Then f is Gβ\*-continuous but none of Gω- continuity and Grg-continuous, since {b} is closed in Y but ({b}) = {b} is none of Gω- closed and Grg-closed in (X, τ,).

**3.8 Example:** Let X = Y = {a, b, c}, τ = {,{a},{b, c},X}, σ = {,{a, c},Y},

= {{b},{a, b}, {b, c}, X} and = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,) (Y, σ, ) be the identity map. Here Gβ\*c(τ)= {, {a},{b, c}, X}. Then f is Gω-continuous and Grg continuous but not Gβ\*-continuous for ({b})={b} is not Gβ\*-closed in (X, τ,).

**3.9 Proposition**: The following examples, show that Gβ\*-continuity is independent of Gg-continuity, Gsg-continuity and Ggs-continuity.

**3.10 Example:** Let X= {a, b, c}=Y , τ={,{a},{b, c},X}, σ={,{b}, Y}, = {{b},{a, b}, {b, c}, X} and = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,) (Y, σ, ) be the identity map. Then f is Gg-continuous, Ggs-continuous and Gsg-continuous, but not Gβ\*-continuous because for the closed set {a, c}, ({a, c}) = {a, c} is not Gβ\*-closed in (X, τ,).

**3.11 Example**: Let X=Y= {a, b, c}, τ= {, {a}, {b}, {a, b}, X}, σ={,{a, c},Y}, = {{b},{a, b}, {b, c}, X} and = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,) (Y, σ, ) be the identity map. Then f is Gβ\*-continuous but none of Gg- continuous, Gsg-continuous and Ggs-continuous because for the closed set {b} in Y, ({b}= {b}is Gβ\*-closed but none of Gg-closed, Gsg-closed and Ggs-closed.

**3.12 Proposition**: The following examples show that Gβ\*-continuity is independent of G-pre-continuity, G-semi-pre-continuity and G-pre-semi-continuity.

**3.13 Example**: Let X=Y= {a, b, c, d}, τ= {, {a, b}, X}, σ= {, {c}, Y}, = {{b},{a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X} and = {{a},{a, b},{a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d}, Y}. Let f: (X, τ,) (Y, σ, ) be the identity map. Then f is Gβ\*-continuous but none of G-pre-continuous, G-semi-pre-continuous and G-pre-semi-continuous, since {a, b, d} is closed in Y but ({a, b, d}) = {a, b, d} is none of G-preclosed, G-semi-preclosed and G-pre-semiclosed in (X, τ,).

**3.14 Example:** Let X= {a, b, c}=Y , τ={,{a},{b, c},X}, σ={,{c}, Y}, = {{b},{a, b}, {b, c}, X} and = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,) (Y, σ, ) be the identity map. Then f is G-pre-continuous, G-semi-pre-continuous and G-pre-semi-continuous, but not Gβ\*-continuous because for the closed set {a, b}, ({a, b}) = {a, b} is not Gβ\*-closed in (X, τ,).

**3.15 Proposition**: The following examples show that Gβ\*-continuity is independent of G\*g-continuity and Gg\*-continuity.

**3.16 Example**: Let X= {a, b, c}=Y, τ = {, {a}, {b}, {a, b}, X}, σ= {, {a, c}, Y}, = {{b}, {a, b}, {b, c}, X} and = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,) (Y, σ, ) be the identity map. Then f is Gβ\*-continuous but neither Gg\*- continuous nor G\*g-continuous because for the closed set {b} is Gβ\*-closed but neither Gg\*-closed nor G\*g-closed in (X, τ,).

**3.17 Example**: Let X= {a, b, c, d}, Y= {a, b, c}, τ = {, {a, b}, X}, σ= {, {a, c}, Y}, ={{a}, {a, b}, {a, c}, {a, d}, {a, b, c}, {a, b, d}, {a, c, d},X} and = {{a},{a, b},{a, c}, Y}. Let f: (X, τ,) (Y, σ, ) be the identity map. Then f is Gg\*- continuous and hence G\*g-continuous but not Gβ\*-continuous because for the closed set {b} is Gg\*-closed and hence G\*g-closed but not Gβ\*-closed in (X, τ,).

From the above discussion are shown in the following implications:

Gβ\*- continuous

G-semi continuous

Gα-continuous

Ggsp-continuous

Grg-continuous

Gω-continuous

G- continuous

Gsg-continuous

Continuous

Ggs-continuous

Gg-continuous

G semi pre-continuous

G pre-continuous

G pre semi-continuous

G\*g-continuous

Gg\*- continuous

**3.18 Proposition**: The composition of two Gβ\*-continuous maps need not be Gβ\*-continuous.

**3.19 Example**: Let X=Y={a, b, c}, Z={p, q}, τ={,{a},{b},{a, b}, X}, σ={, {a}, Y}, η={,{p}, Z} = {{b},{a, b}, {b, c}, X}, = {{a},{a, b}, {a, c}, Y} and = {{p}, Z}. Let f: (X, τ,) (Y, σ, ) be the identity map and define a map g: (Y, σ,) (Z, η, ) by g(a)=g(b)=q and g(c)=p. Then both f and g are Gβ\*- continuous. Consider the closed set A= {q} in (Z, η). For this set (A) = (A)) = {a, b} is not Gβ\*-closed in (X, τ,). Therefore is not Gβ\*-continuous.

**3.20 Proposition**: If f: XY is Gβ\*-continuous and g: YZ is continuous, then their composition : XZ is Gβ\*-continuous.

**Proof:** Clearly follows from definitions.

**3.21 Theorem:** A map f: XY is Gβ\*- continuous if and only if (U) is Gβ\*-open for every open set U in Y.

**Proof:** Let f: XY be Gβ\*-continuous and U be an open set in Y. Then () is Gβ\* closed in X. But () = and so (U) is Gβ\*-open in X. Converse is similar.

**V. CONCLUSION**

In this paper, we have introduced and analyzed Gβ\*-continuous map in grill topological space. Its relation to various other continuous functions are investigated. Also verified the composition of two Gβ\*-continuous maps need not be Gβ\*-continuous.

**REFERENCE:**

1. Al-Omari & Noiri, T 2011, ‘Decomposition of continuity via grills’, Jordan J. Math. Stat., 4(1), 33-46.
2. Antony Rex Rodgio. J & Jessie Theodore, 2013, Notions via - continuous maps in topological space’, International Journal of Science, Engineering and Technology Research,(2), 818-822.
3. Chattopadhyay, K. C & Thron, W. J, 1977, ‘Extensions of closure spaces’, Can. J. Math., 29 (6), 1277-1286.
4. Chattopadhyay, K. C, Njastad, O & Thron, W. J, 1983, ‘Merotopic spaces and extensions of closure spaces’, Can. J. Math., 35 (4), 613-629.
5. Dhananjoy Mandal & Mukherjee, M.N 2012, ‘On a type of generalized closed sets’, Bol. Soc. Paran. Mat. 301, 67-76.
6. Hatir, E & Jafari, S 2010, ‘On some new calsses of sets and a new decomposition of continuity via grills’, J. Adv.Math.Studies, 3 (1), 33-40.
7. Mukherjee & Dhananjoy Mandal, M.N 2012, ‘On a type of generalized closed sets’, Bol. Soc. Paran. Mat. 301, 67-76.
8. Roy, B & Mukherjee, M. N 2007, ‘On a typical topology induced by a grill’, Soochow J. Math, 33 (4), 771-786.
9. Shyamapada Modak, Decompositions of Generalized Continuity in Grill Topological Spaces, Thai Journal of Mathematics Volume 13 (2015) Number 2 : 511–518.
10. Thenmozhi1 P, Kaleeswari M & Maheswari N, ‘Regular Generalized Closed Sets in Grill Topological Spaces’, International Journal of Science and Research (IJSR) ISSN (Online): 2319-7064
11. Thenmozhi P, Kaleeswari, M & Maheswari, N 2017, ‘Strongly g\*- Closed Sets in Grill Topological Spaces’, International Journal of Mathematics Trends and Technology (IJMTT) – Volume 51 Number 1
12. Gomathi Rajakumari, Jessie Theodore, ‘Gβ\*-closed set in grill topological space’, Sarah Research Journal (ISSN 2319 - 5134) (Under review process).
13. Vibin Salim Raj, S, Senthil kumaran, V & Palaniappan, Y 2021, ‘Generalised Semi Closed Sets In Grill Topological Spaces’, Journal of Advance Research in Mathematics and Statistics (ISSN: 2208-2409)